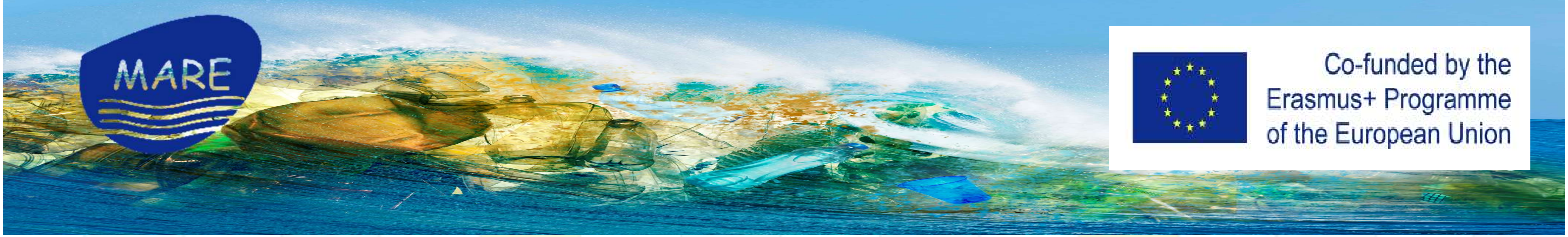

LECTURE 1

MODELLING AND MATHEMATICAL MODELS

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COUSE OUTLINE

1. What is model?
2. What is environmental modelling?
3. Role of environment modelling
4. Types of models
5. Mathematical models?

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1. WHAT IS MODEL?

A Model is a simplification of reality that is constructed to gain insights into select attributes of a physical, biological, economic, or social system. A formal representation of the behaviors of system processes, often in mathematical or statistical terms. The basis can also be physical or conceptual (Environment Protection Agency, 2009)

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2. WHAT IS ENVIRONMENTAL MODELLING ?

Environmental modelling involves the application of multidisciplinary knowledge to explain, explore and predict the Earth’s response to environmental change, both natural and human-induced. (DBW,2018)

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3. ROLE OF ENVIRONMENT MODELLING

- Improved understanding of environmental systems.
- Developing scientific understanding - through quantitative expression of current knowledge of a system (as well as displaying what we know, this may also show up what we do not know);
- Test the effect of changes in a system;
- Aid decision making, including (i) tactical decisions by managers; (ii) strategic decisions by planners.

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4. TYPE OF MODELS

- Physical modeling
- Empirical models
- Mathematical models

4. TYPE OF MODELS

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❖ Physical modeling

- Physical modeling is a way of modeling and simulating systems that consist of real physical components. A physical model is a smaller or larger physical copy of an object.
- Spatial analysis and similarity theories are used in this process to ensure that the model results can be extrapolated to the real system with high accuracy.
- Physical modeling is the main approach of scientists in developing basic theories of the natural sciences.

Water dam model

4. TYPE OF MODELS

❖ Empirical models

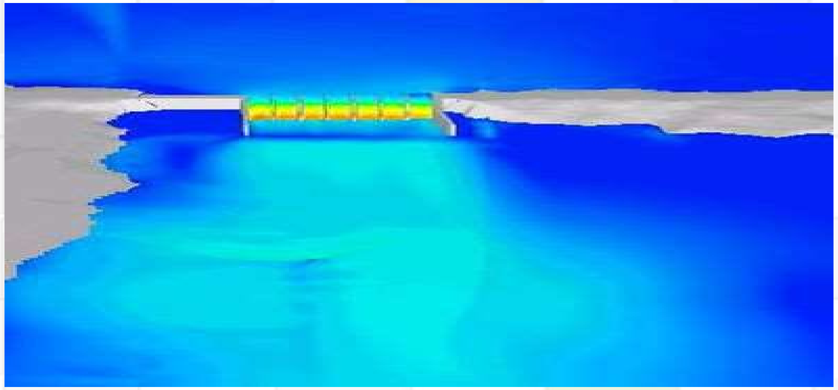
Empirical models describe observed behaviour between variables on the basis of observations alone and say nothing of process. They are usually the simplest mathematical function, which adequately fits the observed relationship between variables. No physical laws or assumptions about the relationships between variables are required. Empirical models have high predictive power but low explanatory depth, they are thus rather specific to the conditions under which data were collected and cannot be generalized easily for application to other conditions

4. TYPE OF MODELS

❖ Mathematical models

- Mathematical model is a representation of real world problem in mathematical form with some simplified assumptions which helps to understand in fundamental and quantitative way
- Mathematical models are much more common and represent states and rates of change according to formally expressed mathematical rules. Mathematical models can range from simple equations to complex software codes applying many equations and rules over time and space discretization. One can further define mathematical models into different types but most models are actually mixtures of many types or are transitional between types.

Dam model is described by a mathematical model



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Real Systems

Mathematic Models

Deterministic Models

Probabilistic models

Continuous

Discrete

Static

Dynamic

Statistical

N=1

N>1

Centralized

Distributed

Markov

Monte Carlo

Algebraic Equations

System of linear equations

Ordinary differential equation

Partial differential equation

Partial differential equation

Linear

Non-Linear

Analytics

Numerical

Classification models

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WHAT IS MATHEMATICAL MODELLING?

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Definition & Meaning

❖ System and boundary

- A system is a set of one or more related objects, which can be a physical entity with specific properties or characteristics. The system is isolated from its surroundings by boundaries, which can be physical or virtual

❖ Open and Closed, flow/non-flow systems

- A closed system is a system that is completely isolated from its environment.
- An open system is a system that has flows of information, energy, and/or matter between the system and its environment, and which adapts to the exchange.
- When the flow of matter does not cross the boundary (but energy can), the system is called a nonflow system. If the material flow can cross the boundary, the system is called a flow system.

Definition & Meaning

❖ **Variable, parameter**

- A variable is a value that changes freely in time and space (a compartment or flow) and a state variable is one which represents a state (compartment). A constant is an entity that does not vary with the system under study, for example, acceleration due to gravity is a constant in most Earth-based environmental models (but not in geophysics models looking at gravitational anomalies, for example).
- A parameter is a value which is constant in the case concerned but may vary from case to case where a case can represent a different model run or different grid cells or objects within the same model.

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APPROACHES TO MATHEMATICAL MODEL BUILDING

Identify problem

↓

System analysis

↓

Choosing mathematic equation

↓

Solving equation

Step to building mathematical models

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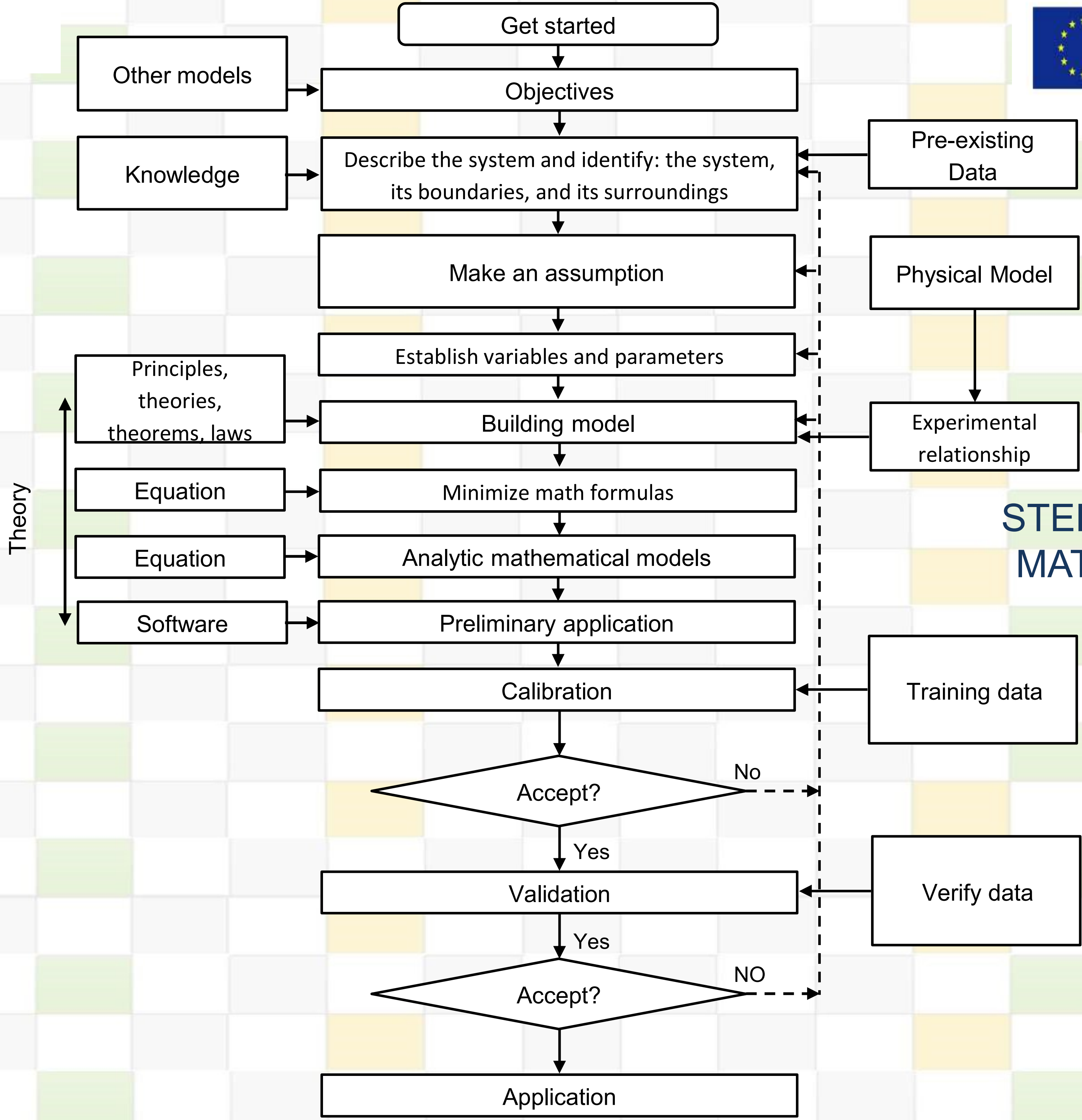
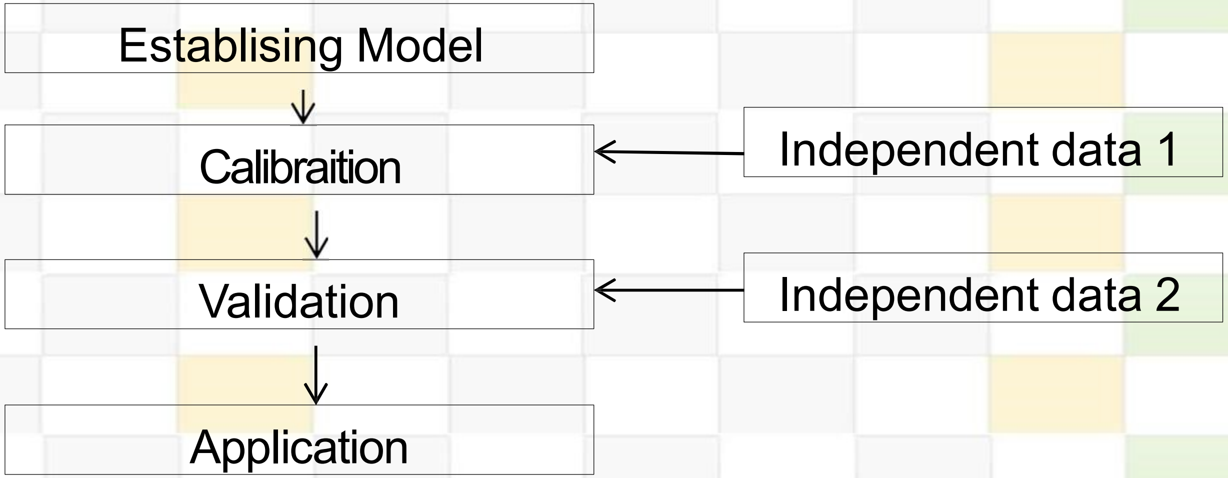
STEP TO BUILDING MATHEMMATICAL MODELS



APPROACHES TO MATHEMATICAL MODEL BUILDING

CALIBRATION AND VALIDATION



- Calibration is **the iterative process of comparing the model with real system**, revising the model if necessary, comparing again, until a model is accepted (validated)
- Validation is a process of comparing the model and its behavior to the real system and its behavior
- Sensitivity analysis is the process of defining how changes in model input parameters affect the magnitude of changes in model output..

APPROACHES TO MATHEMATICAL MODEL BUILDING

Evaluation of simulation results

There are two methods to evaluate model performance

- Graphical method (qualitative)
- Statistical methods (quantitative)

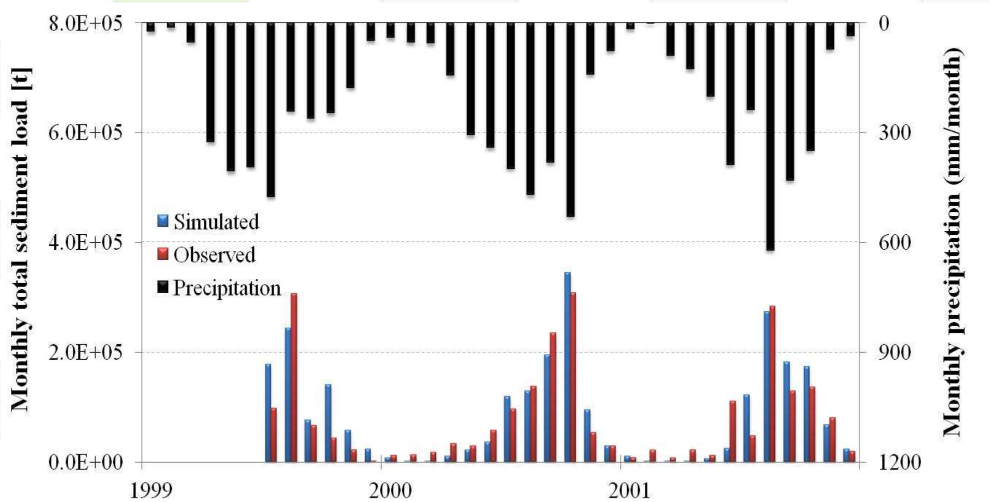
➢ **The calibration model does not represent accurately possibly due to the factors multiply as below:**

- The model is used incorrectly or the model setting is incorrect
- The model is not suitable for this application
- Lack of data to describe the real world
- Measurement data is not reliable

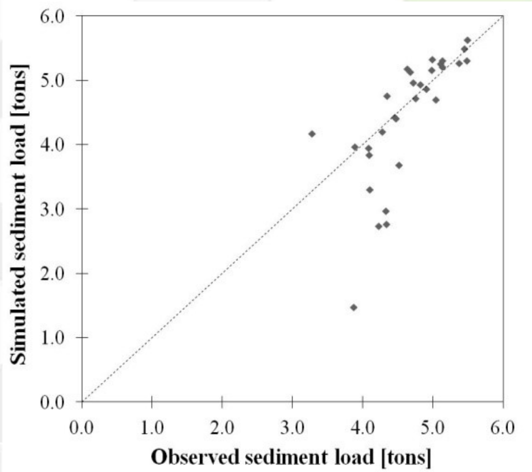
MATHEMATICAL MODEL BUILDING

■ Evaluation of simulation results

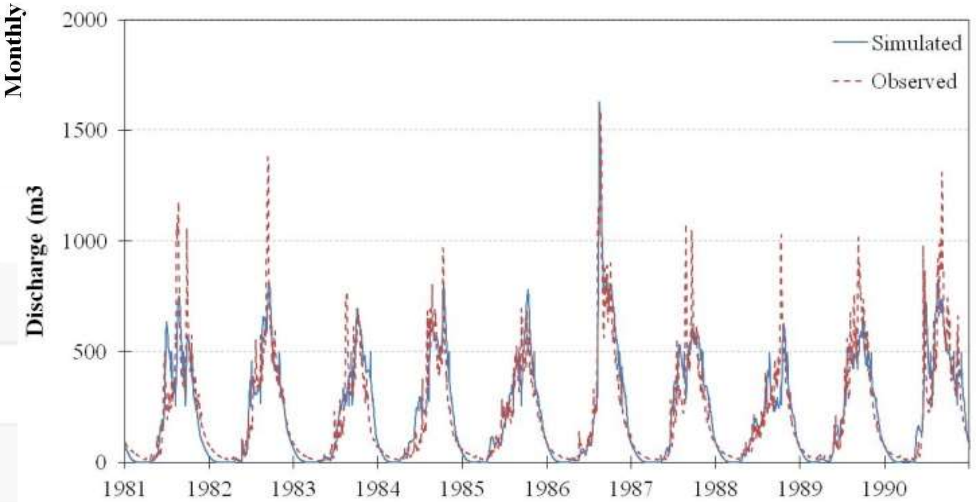
- **Graphical Method**



A. Bar graph



B. Scatter graph



C. Line graph

APPROACHES TO MATHEMATICAL MODEL BUILDING

Evaluation of simulation results (cont.)

- **Statistical method**

Percent bias – PBIAS : measures the average tendency of the simulated values to be larger or smaller than their observed ones.

PBIAS → 0: indicating accurate model simulation.

$$PBIAS = \left[\frac{\sum_{i=1}^{n}\left(Y_i^{obs} - Y_i^{sim}\right) \times 100}{\sum_{i=1}^{n}\left(Y_i^{obs}\right)}\right]$$

Y_i^{obs} : observed value

Y_i^{sim}: Simulate value

n: total observed value / Simulate

APPROACHES TO MATHEMATICAL MODEL BUILDING

Evaluation of simulation results (cont.)

- **Statistical method**

Correlation coefficient formulas R^2 are used to find how strong a relationship is between observed and simulate data.

$$R^2 = \frac{\sum_{i=1}^{n}\left(Y_i^{obs} - \bar{Y}^{obs}\right)\left(Y_i^{sim} - \bar{Y}^{sim}\right)}{\sqrt{\sum_{i=1}^{n}(Y_i^{obs} - \bar{Y}^{obs})^2} \times \sqrt{\sum_{i=1}^{n}(Y_i^{sim} - \bar{Y}^{sim})^2}}$$

$\bar{Y}^{obs}$: average value of the series of observed data

$\bar{Y}^{sim}$: average value of the series of simulated data

APPROACHES TO MATHEMATICAL MODEL BUILDING

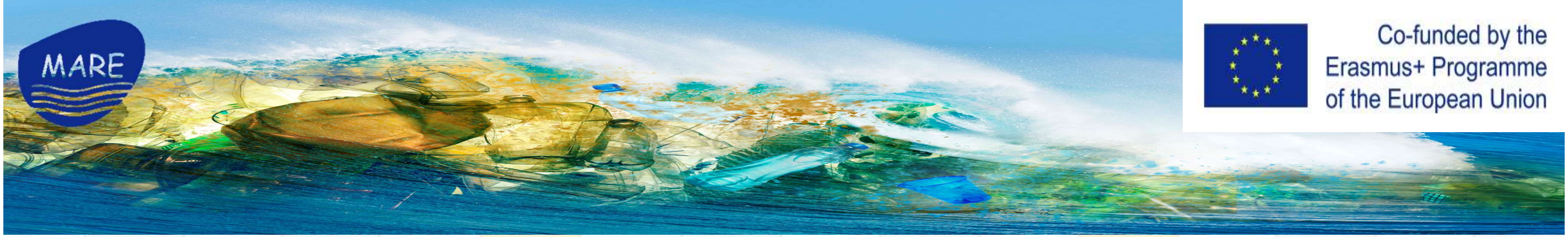
- **Statistical method**

Nash – Sutcliffe (NSE): is a normalized statistic that determines the relative magnitude of the residual variance compared to the measured data variance (Nash and Sutcliffe, 1970). Nash-Sutcliffe efficiency indicates how well the plot of observed versus simulated data fits the 1:1 line. NSE = 1, corresponds to a perfect match of the model to the observed data. NSE = 0, indicates that the model predictions are as accurate as the mean of the observed data, Inf < NSE < 0, indicates that the observed mean is a better predictor than the model.

$$NSE = 1 - \left[\frac{\sum_{i=1}^{n}(Y_i^{obs} - Y_i^{sim})^2}{\sum_{i=1}^{n}(Y_i^{obs} - \bar{Y}^{obs})^2}\right]$$

Properties	NSE, R^2	PBIAS	
		Flow	***Water - quality***
Very Good	0.75 → 1.00	< ± 10 %	<± 25 %
Good	0.65→ 0.75	± 10 % → ± 15 %	± 25 % → ± 40 %
	0.50→0.65	± 15 % → ± 25 %	± 40 % → ± 70 %
Unsatisfactory	<0.50	> ± 25 %	> ± 70 %

(Moriasi et al., 2007)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THE END