LECTURE

MODELLING THE MARINE ENVIRONMENT

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Lecture 5

FUNDAMENTAL EQUATIONS DESCRIBING CURRENTS AND BOUNDARY CONDITIONS

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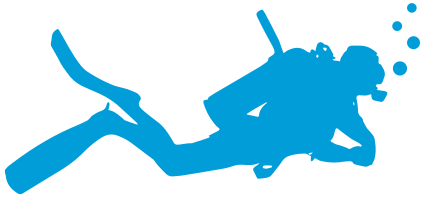

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FUNDAMENTAL EQUATIONS DESCRIBING CURRENTS AND BOUNDARY CONDITIONS

The state of the liquid is completely determined, if at each point of the liquid at any time completely identified:

Pressure P(x,y,z,t);

Density ρ(x,y,z,t),

Velocity with components u(x,y,z,t), v(x, y, z,t), w(x,y,z,t).

Equations of Motion for Perfect and Viscous Fluids

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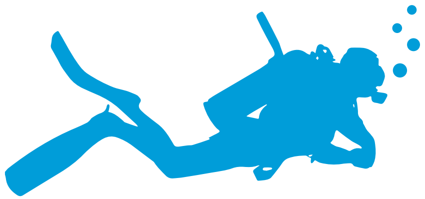
In general, a **particle** is being **acted upon** by the **following forces**

- ✓ **Inertia forces** ($-\gamma = -dV/dt$ for a unit of mass);
- ✓ External forces F (for a unit of mass and with components X,Y,Z)
- ✓ Mutual friction force (pressure, viscous): R.



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Equations of Motion for Perfect and Viscous Fluids

According to the Dalambe principle of the equilibrium of the dx dy dz water particle under the effect of those three forces, we have:

$$\rho \mathrm{dx} \cdot \mathrm{dy} \cdot \mathrm{dz} \cdot \frac{\mathrm{dV}}{\mathrm{dt}} = \rho \cdot \mathrm{F} \cdot \mathrm{dxdydz} + \mathrm{R} \quad (5.1)$$

Projecting the equation (1.25) onto the coordinate axes and dividing by ρ (for non-compressible liquids), we have:

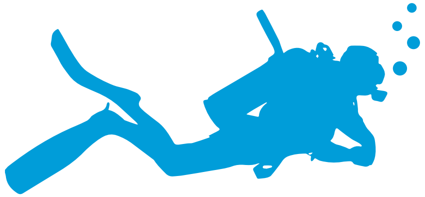
$$\boxed{\begin{aligned} \frac{du}{dt} &= X - \frac{1}{\rho}\frac{\partial P}{\partial x} + \upsilon \nabla u \\ \frac{dv}{dt} &= Y - \frac{1}{\rho}\frac{\partial P}{\partial y} + \upsilon \nabla v \qquad (5.2) \\ \frac{dw}{dt} &= Z - \frac{1}{\rho}\frac{\partial P}{\partial z} + \upsilon \nabla w \end{aligned}}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; υ = const is a kinetic coefficient, for the perfect liquid: v = 0;

$$\begin{aligned} \frac{\mathrm{du}}{\mathrm{dt}} &= \mathrm{X} - \frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial \mathrm{x}} \\ \frac{\mathrm{d}v}{\mathrm{dt}} &= \mathrm{Y} - \frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial \mathrm{y}} \qquad (5.3) \\ \frac{\mathrm{dw}}{\mathrm{dt}} &= \mathrm{Z} - \frac{1}{\rho}\frac{\partial \mathrm{P}}{\partial \mathrm{z}} \end{aligned}$$

Where: $\frac{\mathrm{d}}{\mathrm{dt}}$ is the time differential of a definite liquid particle, and the liquid particle's velovcity is a function of both time and space, so we have:

$$\frac{\mathrm{du}}{\mathrm{dt}} = \frac{\partial \mathrm{u}}{\partial \mathrm{t}} + \mathrm{u}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \mathrm{v}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} + \mathrm{w}\frac{\partial \mathrm{u}}{\partial \mathrm{z}}$$



Equations of Motion for Perfect and Viscous Fluids

Therefore the motion equation has form:

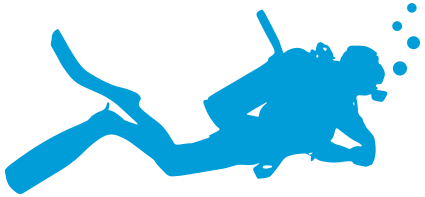
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=X-\frac{1}{\rho}\frac{\partial P}{\partial x}+\upsilon\nabla u$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=Y-\frac{1}{\rho}\frac{\partial P}{\partial y}+\upsilon\nabla v \qquad (5.4)$$
$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=Z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\upsilon\nabla w$$

Equation (5.4) is the Navie-St. equation for the viscous fluids. For the ideal liquid we have:

$$\upsilon\nabla u=\upsilon\nabla v=\upsilon\nabla w=0$$

The above equations are nonlinear. We're going to linearize them if we ignore space acceleration. Movement is considered stable (or stop) if the velocity at any given time does not depend on the time or local acceleration:

$$\frac{\partial u}{\partial t}=\frac{\partial v}{\partial t}=\frac{\partial w}{\partial t}=0$$



CONTINUITY EQUATION

Considering the continuity equation of the fluid, i.e. we consider the continuous properties of that fluid. Assuming there is a fluid volume factor $\delta x\, \delta y\, \delta z$, consider the volume of liquid entering and exiting this volume during the period δt.

In the direction of the Ox axis, the mass of fluid entering that volume: $(\rho u)_x \delta t \delta y \delta z$

and the mass of fluid that comes out of volume is: $(\rho u)_{x+\delta x} \delta t \delta y \delta z$

So, after the period δt $\delta x\, \delta y\, \delta z$ là: Ox axis movement will increase the amount of fluid in the volume factor

$$(\rho u)_x \delta z \delta y \delta t - (\rho u)_{x+\delta x} \cdot \delta z \delta y \delta t = -\frac{\partial(\rho \cdot u)}{\partial x} \delta x \delta y \delta z \delta t$$

Similarly, in the direction of the Oy axis and the Oz axis we also have:

$$-\frac{\partial(\rho . v)}{\partial y} \delta x \delta y \delta z \delta t; -\frac{\partial(\rho \cdot w)}{\partial z} \delta x \delta y \delta z \delta t$$

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CONTINUITY EQUATION

According to the law of mass conservation, the total volume of liquids entering and exiting volume δx δy δz must be by changing the volume of fluid during that time.:

$$\left(\rho+\frac{\partial\rho}{\partial \mathrm{t}}\partial \mathrm{t}\right)\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}-\rho\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}=\frac{\partial\rho}{\partial \mathrm{t}}\delta \mathrm{t}\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}$$

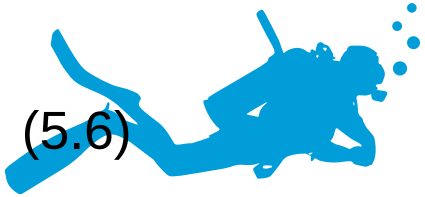
$$=-\frac{\partial(\rho \mathrm{u})}{\partial \mathrm{x}}\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}\mathrm{t}-\frac{\partial(\rho \mathrm{v})}{\partial \mathrm{y}}\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}\delta \mathrm{t}-\frac{\partial(\rho \mathrm{w})}{\partial \mathrm{z}}\delta \mathrm{x}\delta \mathrm{y}\delta \mathrm{z}\delta \mathrm{t}$$

$$\frac{\partial\rho}{\partial \mathrm{t}}+\frac{\partial(\rho \mathrm{u})}{\partial \mathrm{x}}+\frac{\partial(\rho \mathrm{v})}{\partial \mathrm{y}}+\frac{\partial(\rho \mathrm{w})}{\partial \mathrm{z}}=0$$

Or
$$\frac{\partial\rho}{\partial \mathrm{t}}+\mathrm{div}(\rho.\mathrm{V})=0 \qquad (5.5)$$

The equation (5.5) is the continuity equation of compressed liquid. In practical calculations it is common to view liquids as uncompressed ρ = const: $\frac{\partial\rho}{\partial \mathrm{t}}=0$

Continuity equation is constantly in form: $\mathrm{div}V=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ (5.6)



SALT CONSERVATION EQUATION

Consider the amount of salt that goes beyond the limits of a volume factor $\delta x\ \delta y\ \delta z$ follow the Ox axis at a time of ration x during the period δt là $(S.\rho.u)_x\ \delta y\delta z\delta t$, The amount of salt passing through the opposite side is $(S.\rho.u)_{x+\delta x}\delta y\delta z\delta t$.

Therefore, the movement in the direction of the Ox axis has the excess salt.: $-\frac{\partial(\rho\cdot u.S)}{\partial x}\delta x\delta yz\delta t$

In the direction of the axes Oy and Oz also: $\frac{\partial(\rho\cdot u.S)}{\partial y}\delta x\delta y\delta z\delta t$; $-\frac{\partial(\rho\cdot w\cdot S)}{\partial z}\delta x\delta y\delta z\delta t$

Order the initial amount of salt in volume δx δy δz là ρS.δx δy δz, after δt the amount of salt in the volume factor is: $\left(\rho\cdot S+\frac{\partial(\rho\cdot S)}{\partial t}\delta t\right)\delta x\delta y\delta z$

According to the law of conservation: $+\frac{\partial}{\partial t}(\rho\cdot u\cdot S)+\frac{\partial}{\partial y}(\rho\cdot v\cdot S)+\frac{\partial}{\partial z}(\rho\cdot w\cdot S)=0$ (5.7)

When using continuity equations (1.29) we have: $\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}+w\frac{\partial S}{\partial t}=0$ (5.8)

When S(x,y,z) = const , so $\frac{\partial S}{\partial t}=0$, therefore: $u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}+w\frac{\partial S}{\partial z}=0$ (5.9)

The heat conservation equation is received in the same way.:

$$\frac{\partial T}{\partial t}+\mathbf{u}\frac{\partial T}{\partial x}+\mathbf{v}\frac{\partial T}{\partial y}+\mathbf{w}\frac{\partial T}{\partial z}=\mathbf{0} \qquad \mathbf{(5.10)}$$

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SEAWATER STATE EQUATION

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Seawater is a compressed liquid, i.e. its density changes. The dependence of the specific volume α and the density of the water on the state parameters: T temperature, S salinity and P pressure are indicated by the state equation. The general form of the state equation is as follows:

$$\rho = \rho(\mathrm{T}, \mathrm{S}, \mathrm{P}) \qquad (5.11)$$

$$\alpha = \alpha(\mathrm{T}, \mathrm{S}, \mathrm{P}) \qquad (5.12)$$

Determines the specific volume change of seawater as a function of state parameters:

$$\mathrm{d}\alpha = \left(\frac{\partial\alpha}{\partial \mathrm{T}}\right)_{\mathrm{SP}} \mathrm{dT} + \left(\frac{\partial\alpha}{\partial \mathrm{S}}\right)_{\mathrm{TP}} \mathrm{dS} + \left(\frac{\partial\alpha}{\partial \mathrm{P}}\right)_{\mathrm{TS}} \mathrm{dP}. \qquad (5.13)$$

If you divide all the components of (1.37) by a particular unit volume α_0 the pre-fractions of temperature, salt and pressure will be:

- Thermal expansion coefficient:

$$\mathrm{K_T} = \frac{1}{\alpha_0}\left(\frac{\partial\alpha}{\partial \mathrm{T}}\right)_{\mathrm{S.P}} \qquad (5.14)$$

- Salt compression coefficient:

$$\mathrm{K_s} = -\frac{1}{\alpha_0}\left(\frac{\partial\alpha}{\partial \mathrm{S}}\right)_{\mathrm{T.P}}$$

- Compression resistance coefficient of density:

$$\mathrm{K_p} = -\frac{1}{\alpha_0}\left(\frac{\partial\alpha}{\partial \mathrm{P}}\right)_{\mathrm{T.S}}$$

SEAWATER STATE EQUATION

Then the expression (5.14) is often called the equation of state in the differential form of seawater

$$\frac{d\alpha}{\alpha_0} = K_T dT - K_s dS - K_P dP \qquad (5.15)$$

Currents theories often use simpler systems. The simplest of these is the Businesq approximation (the linear dependence of density on temperature): $\frac{\rho}{\rho_0} = C_1 + C_2 \frac{T}{T_0}$ (5.16)

and the linear dependence of density on temperature and salt (Linheikin, Robinson and Stommel, Bryan and Kox):

$$\frac{\rho}{\rho_0} = C_3 + C_4 \frac{T}{T_0} + C_5 \frac{S}{S_0}. \qquad (5.17)$$

Where T is temperature, S is salt, ρ_0 is the average density of seawater; T_0 and S_0 are the average values of temperature and salt. When atmospheric pressure is equal 1 at, T_0=17,5^{0}C, S_0 = 35$^0/_{00}$,

ρ_0 = 1,02541 g/cm^3 thì các hệ số có giá trị: C_1 = 1,00266; C_2 = C_4 = -0,00317; C_3 = 0,97529; C_5 = 0,02737.

More precise dependence of density on temperature and salt: $\frac{\rho}{\rho_0} = C_6 + C_7 \frac{T}{T_0} + C_8 \frac{S}{S_0} + C_9 \left(\frac{T}{T_0}\right)^2$ (5.18)

with values T_0, S_0, ρ_0 and atmospheric pressure as above C_6 = 0,97529, C_7= - 0,00006, C_8 = 0,02737, C_9 = - 0,0014.

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SEAWATER STATE EQUATION

If the compression of the liquid, i.e. the variation of density and the pressure ratio is taken into account, there are:

$$\frac{\rho}{\rho_0} = \left[C_6 + C_7\frac{T}{T_0} + C_8\frac{S}{S_0} + C_9\left(\frac{T}{T_0}\right)^2 + C_{10}\frac{T}{T_0}\frac{S}{S_0}\right] \times \left[1 + \frac{P - P_0}{P_0}C_{11}\right] \quad (5.19)$$

Where P_0 is pressure is equal to 1 at., C_{11} is a constant quantity and a compression-resistant coefficient, can take: C_{10} = - 0,00119, C_{11} = 0,428.10^{-4}.

If you view the pressure as proportional to the depth, the equation (5.19) has:

$$\frac{\rho}{\rho_0} = \left[C_6 + C_7\frac{T}{T_0} + C_8\frac{S}{S_0} + C_9\left(\frac{T}{T_0}\right)^2 + C_{10}\frac{T}{T_0}\frac{S}{S_0}\right] \times \left[1 + C_{12}\frac{Z}{Z_0}\right] \quad (5.20)$$

And simpler dependency(5.20) is:

$$\frac{\rho}{\rho_0} = \left[C_6 + C_7\frac{T}{T_0} + C_8\frac{S}{S_0} + C_9\left(\frac{T}{T_0}\right)^2 + C_{10}\frac{T}{T_0}\frac{S}{S_0}\right] + C_{13}\frac{Z}{Z_0} \quad (5.21)$$

Where Z0 = 1km, it is possible to take C12 = 0.00428, C13 = 0.0043; (1.44) and (1.45) by Linheikin, Mamaev, Vaxilev. Equations (5.19) and (5.20), although not allowing for accurate calculation of density, have been used to solve most of the problems of sea current theory that involve examining the nonlinear interactions of the fields of flow velocity, density, temperature, and salt level.

TURBULENT MOTION, REYNOLDS STRESS

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The system of moving equations, continuous, state and preservation of salt heat is closed. But to get accurate results from solving those equations is impossible because the movement of the real liquid always has a tangled feature. Therefore, we must consider the tangled characteristics in these equations. Performs real motion in the form of medium motion and sublimation motion:

$$\mathrm{u} = \bar{u} + \mathrm{u}'; \mathrm{v} = \bar{v} + \mathrm{v}'; \mathrm{w} = \bar{w} + \mathrm{w}'$$

$$\mathrm{P} = \bar{P} + \mathrm{P}'; \mathrm{T} = \bar{T} + \mathrm{T}'; \mathrm{s} = \bar{s} + \mathrm{s}'; \rho = \bar{\rho} + \rho$$

Reynolds' 5 conditions for any function: $q_1 = \bar{q}_1 + q'_1$; $q_2 = \bar{q}_2 + q'_2$;

$$q_1 + q_2 = \bar{q}_1 + \bar{q}_2 \qquad (5.22)$$

$$\overline{aq_1} = \overline{aq}, \text{khi } a = \text{const} \qquad (5.23)$$

$$\bar{a} = a, \text{khia} = \text{const} \qquad (5.24)$$

$$\overline{\frac{\partial q_1}{\partial x_i}} = \frac{\partial \overline{q_1}}{\partial x_i}; \overline{\frac{\partial q_1}{\partial t}} = \frac{\partial \overline{q_1}}{\partial t}; \qquad (5.25)$$

$$\overline{\overline{q_1} \cdot q_2} = \overline{q_1} \cdot \overline{q_2} \qquad (5.26)$$

$$\overline{\overline{q}_1} = \bar{q}_1 \qquad (5.27)$$

$$\overline{q'_1} = 0 \qquad (5.28)$$

$$\overline{\overline{q_1} \cdot \overline{q_2}} = \overline{q_1} \cdot \overline{q_2} \qquad (5.29)$$

$$\overline{\overline{q_1} \cdot q'_2} = 0 \qquad (5.30)$$

When considering the velocity field we have the average value of speed pulses over a certain period of time will be zero:

$$\overline{u'} = \frac{1}{T}\int_t^{t+T} u' dt = 0 \qquad (5.31)$$

TURBULENT MOTION, REYNOLDS STRESS

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MOTION EQUATION IN TURBULENT MOTION

At any time and points, the speed components must satisfy the Navie-Stoc equation, so according to the Ox axis we have:

$$\frac{\partial\bar{u}}{\partial t}+\frac{\partial u'}{\partial t}+(\bar{u}+u')\left(\frac{\partial\bar{u}}{\partial x}+\frac{\partial u'}{\partial x}\right)+(\bar{v}+v')\left(\frac{\partial\bar{u}}{\partial y}+\frac{\partial u'}{\partial y}\right)+(w'+\bar{w})\left(\frac{\partial\bar{u}}{\partial z}+\frac{\partial u'}{\partial z}\right)=X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}-\frac{1}{\rho}\frac{\partial P'}{\partial x}+\upsilon\Delta\bar{u}+\upsilon\Delta u' \quad (5.32)$$

According to the Reynolds systems and the consequences we have:

$$\overline{u'}=0;\ \overline{\frac{\partial u'}{\partial x}}=0;\ \overline{\frac{\partial u'}{\partial t}}=0 \qquad (5.33)$$

When we take the equation average (5.32) over the T period, we have:

$$\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}+\overline{u'\frac{\partial u'}{\partial x}}+\overline{v'\frac{\partial u'}{\partial y}}+\overline{w'\frac{\partial u'}{\partial z}}=X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}+\upsilon\Delta\bar{u} \qquad (5.34)$$

Hay: $\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}=X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}-v\Delta\bar{u}-\frac{\overline{u'\partial u'}}{\partial x}-\frac{\overline{v'\hat{u}'}}{\partial y}-\frac{\overline{w'\partial u'}}{\partial z}=X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}+v\overline{\Delta u}-\frac{\overline{u'^2}}{\partial x}-\frac{\overline{\partial u'v'}}{\partial y}-\frac{\overline{\partial u'w'}}{\partial z}+u'\frac{\partial u'}{\partial x}+u'\frac{\partial v'}{\partial y}+u'\frac{\partial w'}{\partial z}$

When using continuous equations for medium and pulse motion and viewing the liquid as uncompressed, we have:

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}=\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0$$

Therefore: $\frac{\partial\bar{u}}{\partial t}+\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}=X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}=\upsilon\Delta\bar{u}-\frac{\overline{\partial u'^2}}{\partial x}-\frac{\overline{\partial u'v'}}{\partial y}-\frac{\overline{\partial u'w'}}{\partial z}$

$$X-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x}+\frac{1}{\rho}\left(\varepsilon\Delta\bar{u}-\rho\frac{\overline{\partial u'^2}}{\partial x}-\rho\frac{\overline{\partial u'v'}}{\partial y}-\rho\frac{\overline{\partial u'w'}}{\partial z}\right) \qquad (5.35)$$

MOTION EQUATION IN TURBULENT MOTION

Thus the Navie-Stoc equation with tangled motion has been taken on average different from the previous equation of having additional nonlinear components.:

$$-\frac{\partial\overline{(\mathrm{u}'^2)}}{\partial \mathrm{x}}-\frac{\partial\overline{(\mathrm{u}'\mathrm{v}')}}{\partial \mathrm{y}}-\frac{\partial\overline{(\mathrm{u}'\mathrm{w}')}}{\partial \mathrm{z}}$$ theo trục Ox;

$$-\frac{\partial\overline{(\mathrm{u}'\mathrm{v}')}}{\partial \mathrm{x}}-\frac{\partial\overline{(\mathrm{v}'^2)}}{\partial \mathrm{y}}-\frac{\partial\overline{(\mathrm{u}'\mathrm{w}')}}{\partial \mathrm{z}}$$ theo trục Oy;

$$-\frac{\partial\overline{(u'w')}}{\partial x}-\frac{\partial\overline{(v'w')}}{\partial y}-\frac{\partial\overline{(w'^2)}}{\partial z}$$ theo trục Oz;

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Motion Equation In Turbulent Motion

Similar to the viscous case of the element viscosity we have:

$$-\overline{\rho u'^2} = 2\varepsilon \frac{\partial \bar{u}}{\partial x} = \tau_{xx}$$

$$-\overline{\rho u'v'} = \overline{\rho v'u'} = \varepsilon \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right) = \tau_{yx} = \tau_{xy}$$

$$-\overline{\rho u'w'} = \overline{\rho w'u'} = \varepsilon \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right) = \tau_{zx} = \tau_{xz} \qquad (5.36)$$

Với $\varepsilon = \upsilon\rho$

$$-\overline{\rho v'^2} = 2\varepsilon \frac{\partial \bar{v}}{\partial y} = \tau_{yy}$$

$$-\overline{\rho v'w'} = \overline{\rho w'v'} = \varepsilon \left(\frac{\partial v}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right) = \tau_{zy} = \tau_{yz}$$

$$-\overline{\rho w'^2} = 2\varepsilon \frac{\partial \bar{w}}{\partial z} = \tau_{zz}$$

It is the component of reynolds tenxo or tangled tenxo of tangled motion:

$$\begin{pmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{pmatrix}$$

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Motion equation in turbulent motion

If you consider flat movement, the movement is constant. $\frac{du}{dt} = 0$ have a speed (u) parallel to the Ox axis and depend only on z, there are:

$$0 = X - \frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \frac{1}{\partial}\left(\varepsilon \frac{d^2\bar{u}}{\partial z^2} - \rho \frac{d\overline{u'w'}}{dz}\right)$$

$$0 = X - \frac{1}{\rho}\frac{\partial \bar{P}}{\partial x} + \frac{1}{\partial}\frac{d}{dz}\left(\varepsilon \frac{d\bar{u}}{\partial z} - \rho \bar{u}'w'\right) \qquad (5.37)$$

Similar to element friction, to find the turbulent viscosity coefficient μ we write the pressure in the form of:

$$F = \mu \frac{d\bar{u}}{dZ} = \varepsilon \frac{d\bar{u}}{dz} - \rho \overline{u'w'}$$

Because in **turbulent** motion. $\varepsilon \frac{d\bar{u}}{dz}$ very small compared to $\rho \overline{u'w'}$ So we can skip and watch closely: $F = \mu \frac{d\bar{u}}{dz} = -\rho \overline{u'w'}$

Therefore $$\mu = -\rho \frac{\overline{u'w'}}{\left(\frac{du}{dx}\right)} \qquad (5.38)$$

It's a formula for calculating a turbulent viscosity coefficient. Theo Businesq the coefficient μ It depends mainly on the intensity of the disorder. The turbulent viscosity coefficient has a range from 101 - 103 CGS.

TURBULENT MOTION, REYNOLDS STRESS

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Motion equation in turbulent motion

The assertion of u'v', u'w', v'w' in the other mean line does not allow to say that there must be some correlation between the ascension u', v', w'. When there is no correlation, u'v' is Zero. The correlation coefficient is calculated as follows:

$$K = \frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}.\sqrt{\overline{v'^2}}} \qquad (5.39)$$

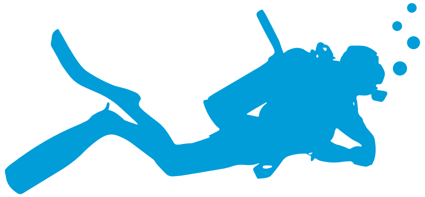
It has been determined that when the density is highly stable, the slime coefficient is small. So when the water mass has great stability, the Reynolds stress contains the w' component that must be smaller than reynolds stress without w', so the horizontal and vertical viscosity coefficients are different.

$$\tau_{xx} = 2\mu_V \frac{\partial \bar{u}}{\partial x} = -\rho\overline{u'^2} \qquad \tau_{zz} = 2\mu_h \frac{\partial \bar{w}}{\partial z} = -\rho\overline{w'^2} \qquad \tau_{xz} = \tau_{zx} = \mu_h \frac{\partial \bar{u}}{\partial z} + \mu_V = -\rho\overline{u'v'}$$

$$\tau_{yy} = 2\mu_V \frac{\partial \bar{v}}{\partial y} = -\rho\overline{v'^2} \qquad \tau_{xy} = \tau_{yx} = \mu_V\left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\right) = -\rho\overline{u'v'} \qquad \tau_{xy} = \tau_{yx} = \mu_V\left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\right) = -\rho\overline{u'v'} \qquad (5.40)$$

- $\mu_{h:}$ the turbulent viscosity coefficient of the horizontal speed gradient

- μ_v the turbulent viscosity coefficient of the vertical speed gradient.



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Motion equation in turbulent motion

If the pressure pressures act on a fluid volume factor. δx, δy, δz, with hypothesis μ_h và μ_v is constant and considering the continuity equation for average motion, there are:

$$\delta x \delta y \delta z\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)=\delta x \delta y \delta z\left(\mu_h \frac{\delta^2 \bar{u}}{\partial x^2}+\mu_h \frac{\partial^2 \bar{u}}{\partial y^2}+\mu_h \frac{\partial^2 \bar{u}}{\partial z^2}\right)$$

The average motion equation is rewritten as:

$$\frac{d\bar{u}}{dt}=\frac{\partial \bar{u}}{\partial t}+u\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}=X-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\frac{\mu_v}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2}+\frac{\partial^2 \bar{u}}{\partial y^2}\right)+\frac{\mu_h}{\rho}\frac{\partial^2 \bar{u}}{\partial z^2}$$

$$\frac{d\bar{v}}{dt}=\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial x}+\bar{w}\frac{\partial \bar{v}}{\partial z}=Y-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y}+\frac{\mu_v}{\rho}\left(\frac{\partial^2 \bar{v}}{\partial x^2}+\frac{\partial^2 \bar{v}}{\partial y^2}\right)+\frac{\mu_h}{\rho}\frac{\partial^2 \bar{v}}{\partial z^2}$$

$$\frac{d\bar{w}}{dt}=\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}=X-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z}+\frac{\mu_v}{\rho}\left(\frac{\partial^2 \bar{w}}{\partial x^2}+\frac{\partial^2 \bar{w}}{\partial y^2}\right)+\frac{\mu_h}{\rho}\frac{\partial^2 \bar{w}}{\partial z^2}) \qquad (5.41)$$

Coefficients $\mu_h \mu_v$ determined from Reynolds stress. Prandtl gives Reynolds stress formula as follows:

$\tau_{zx}=-\rho\,\overline{\mu' w'}\left|\frac{d\bar{u}}{dz}\right|\frac{d\bar{u}}{dz}$ (5.42) L is a turbulent road and $\mu=\rho\ell^2\frac{d\bar{u}}{dz}$ (5.43)

Thus the coefficient of μ proportional to the cube of the turbulent distance.

TURBULENT MOTION, REYNOLDS STRESS

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Salt Conservation Equation in turbulent motion

We have: $$\left[\frac{\partial}{\partial t} + (\bar{u} + u')\frac{\partial}{\partial x} + (\bar{v} + v')\frac{\partial}{\partial y} + (\bar{w} + w')\frac{\partial}{\partial t}\right](\bar{S} + S') = 0 \qquad (5.44)$$

If the liquid to be uncompressed,: $$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

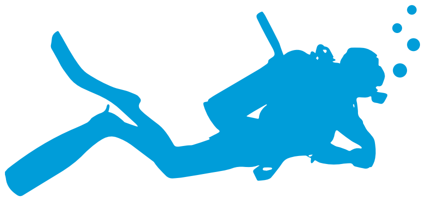
And take the average equation (5.44) we have: $$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial t} + \bar{w}\frac{\partial}{\partial t}\right)\bar{S} + \frac{\partial}{\partial x}\overline{u'S'} + \frac{\partial}{\partial u}\overline{v'S'} + \frac{\partial}{\partial z}\overline{w'S'} = 0 \qquad (5.45)$$

In the concept of a turbulent diffusion coefficient: Ax, Ay, Az of salt in the directions Ox, Oy, Oz. These coefficients are calculated according to expressions: $\overline{u's'} = -\frac{A_x}{\rho}\frac{\partial \bar{S}}{\partial x}; \overline{v's'} = -\frac{A_y}{\rho}\frac{\partial \bar{S}}{\partial y}; \overline{w's'} = -\frac{A_z}{\rho}\frac{\partial \bar{S}}{\partial z}$ (5.46)

Hence, . $$\frac{\partial S}{\partial t} + \bar{u}\frac{\partial S}{\partial x} + \bar{v}\frac{\partial S}{\partial y} + \bar{w}\frac{\partial S}{\partial z} = \frac{\partial}{\partial x}\left(\frac{A_x}{\rho}\frac{\partial S}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{A_y}{\rho}\frac{\partial S}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{A_z}{\rho}\frac{\partial S}{\partial z}\right) \qquad (5.47)$$

(5.47) is the equation that diffuses salt in the sea. Similarly, we also found a thermal diffusion equation.

$$\frac{\partial \bar{T}}{\partial t} + \bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} + \bar{w}\frac{\partial \bar{T}}{\partial z} = \frac{\partial}{\partial x}\left(\frac{A_{Tx}}{\rho}\frac{\partial \bar{T}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{A_{Ty}}{\rho}\frac{\partial \bar{T}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{A_{Tz}}{\rho}\frac{\partial \bar{T}}{\partial z}\right) \qquad (5.48)$$



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BOUNDARY CONDITIONS

Equations of motion are differential equations, in order to solve those equations, there must be boundary conditions or limit conditions. Boundary conditions are generally divided into three categories:

1. Dynamic boundary conditions.

2. Kinetic boundary conditions.

3. Thermal and salt conditions.

TURBULENT MOTION, REYNOLDS STRESS

Dynamic boundary conditions

These are conditions that indicate the continuity of the stressor at the boundary between the atmosphere and the ocean.

When z = - ζ (x,y,t), That is, on the free side of the ocean.:

Where Pa is the atmospheric pressure $$\mathrm{P} = \mathrm{P_a} \qquad (5.49)$$

$$\mu_h \frac{\partial u}{\partial z} = -\tau_x; \ \mu_h \frac{\partial v}{\partial z} = -\tau_y \qquad (5.50)$$

where as τ_x, τ_y is a wind tangential stress on the sea surface.

Because of the lowering of ocean levels ζ It is usually very small compared to the depth of the sea, so these conditions are sometimes given on the non-noisy side of the sea.: z = 0.

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Kinetic boundary conditions

This condition indicates impermeability to liquids on the free side:

z = - ζ (x, y, t), tại đáy z = H (x,y) and in boundaries.

- Khi z= - ζ(x, y, t) $$W = -\frac{d\xi}{dt} = -\left(\frac{d\xi}{dt} + u\frac{d\xi}{dx} + v\frac{d\xi}{dy}\right) \qquad (5.51)$$

- Khi z = H(x,y): Kinetic boundary condition can take two forms.:

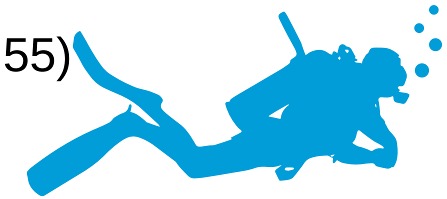
a) $W = u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}$ frictionless sliding conditions (5.52)

b) $u = v = 0, w = 0$ no-slip condition (5.53)

The choice of condition a) or b) is dependent on friction with the bottom. Sliding conditions do not take into account the bottom border layer.

- At the solid boundary: u = v = 0 adhesive and impermeable boundary conditions (5.54)

- At the liquid boundary, give before the distribution of the speed vector: $V_L = V_L(x, y, z)$ (5.55)



Thermal and salt conditions

These conditions denote the effect of the transport of thermal and salt through the dividing sides.

On the free side of the ocean.: Khi z = - ζ(x,y,t) the general form of these conditions is:

$$\gamma \mathrm{T} + \delta \frac{\partial \mathrm{T}}{\partial \mathrm{z}} = \mathrm{G_T} \qquad\qquad \gamma \mathrm{S} + \delta \frac{\partial \mathrm{S}}{\partial \mathrm{z}} = \mathrm{G_S} \qquad (5.56)$$

if δ = 0 The condition is for the values of the function itself., and if γ = 0 then for the gradient of that function.

- At the bottom and at the lateral solid boundaries, for conditions without the flow of thermal and salt according to the route method with the margin:

$$\frac{\partial \mathrm{T}}{\partial \mathrm{n}} = \frac{\partial \mathrm{S}}{\partial \mathrm{n}} = 0 \qquad (5.57)$$

- At the side fluids boundaries: $\frac{\partial \mathrm{T}}{\partial \mathrm{n}} = \mathrm{G_{Tn}}; \frac{\partial \mathrm{S}}{\partial \mathrm{n}} = \mathrm{G_{Sn}}$ (5.58)

The initial conditions reflect the thermal state of the ocean at the initial time t = 0. It is usually required to give in advance the field of ocean features at the initial time:

$$\mathrm{u} = \mathrm{u}^{(0)}, \mathrm{v} = \mathrm{v}^{(0)}, \mathrm{w} = \mathrm{w}^{(0)}, \mathrm{p} = \mathrm{p}^{(0)}, \mathrm{T} = \mathrm{T}^{(0)}, \mathrm{s} = \mathrm{s}^{(0)}, \rho = \rho^{(0)} \qquad (5.59)$$

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Classification of non-stop processes in the ocean and some approximations applied to currents research

When studying the ocean, there are seeing the phenomenon of fluctuations over time of marine fields such as the velocity field., temperature field T^0, salt degree field $S^0/_{00}$, density ρ..., they make up a variety of physical processes in the ocean. To classify these processes in time and space, similar to the classification of changes in climate fields, one derives from the spectrum of the cycle, which divides them into seven time periods.

CLASSIFICATION OF NON-STOP PROCESSES

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1. Small-scale phenomena: Cycles from a few seconds to tens of minutes.
2. Medium-scale phenomena: Cycles from a few hours to daily.
3. Syn scale change: Cycles a few days to months
4. Season fluctuations: Five-year cycles and larger.
5. Changes between years: I.e. changes consistent with the state of large seas and of the entire atmosphere from year to year.
6. Changes in the century: Cycles of several decades. It is the study of the connection between the ocean and changes in the century of climate. \
7. Changes between centuries: Cycles of hundreds of years and larger. It is the study of the connection between the ocean and the fluctuations between the centuries of climate.

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SOME APPROXIMATIONS APPLIED TO CURRENTS RESEARCH

For average movements, the following approximations are correct.:

1. **QUASI-STATIC APPROXIMATION**
2. **APPROX. BUSINESQ**
3. **APPROXIMATIONS TO THE CORIOLIS FORCE**
4. **GEOLOCATION SYSTE**

QUASI-STATIC APPROXIMATION

The studies of medium and large-scale processes in the ocean (vertical scale H ≈ 100 m ÷ 1 km and horizontal scale (L ≈ 100 ÷ 1000 km) show that vertical velocity is much smaller than horizontal velocity. Consider the order of quantity in the conservation of mass equation (the continuity equation):

$$\mathrm{W} = \mathrm{H.U/L} \quad \text{suy ra } \mathrm{W} = 10^{-3}\mathrm{U} \quad (5.60)$$

Where W, U are the characteristic quantities of the vertical and horizontal speeds.

Since the vertical velocity in the ocean is very small, it is possible to write the equation of vertical motion as

$$\frac{\partial P}{\partial z} = g \cdot \rho \qquad (5.61)$$

like the static equation.

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APPROX. BUSINESQ

We know that the density of water in the ocean changes very little : $\frac{\partial\rho}{\rho} \approx 10^{-3}$($\delta\rho$ is the density anomaly), so density ρ can be replaced by ρ_0 (average density), then the equation for conservation of mass is written as: $\mathrm{div}\overrightarrow{\mathrm{V}} = 0$ (5.62)

(incompressible condition of seawater).

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APPROXIMATIONS TO THE CORIOLIS FORCE

When studying medium- and large-scale motion in the ocean as known /W/ <</U/, the term with coefficient $2\omega w\cos\phi$ in the component of the Coriolis force along the Ox axis can be ignored. But it may be necessary to account for this term in the narrow band at the equator.