

LECTURE

MODELLING THE MARINE ENVIRONMENT

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Lecture 6

THEORY OF CURRENTS

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GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES IN FLUIDS

In the conditions that (1) frictionless horizontal flow at a constant speed, (2) the single external force (gravity), and (3) no vertical movement, the horizontal components of Coriolis force and gradient pressure are balanced:

$$2\omega \sin \varphi v = \frac{1}{\rho} \frac{\partial P}{\partial x} \quad (6.1)$$

$$2\omega \sin \varphi u = -\frac{1}{\rho} \frac{\partial P}{\partial y}.$$

If we take the cubes of each equation and add them together, we have:

$$\frac{\partial P}{\partial n} = 2\omega \rho c \sin \varphi \quad (6.2)$$

Ở đây $c = (u^2 + v^2)^{1/2}, \frac{\partial P}{\partial n} = \left[\left(\frac{\partial P}{\partial x} \right)^2 + \left(\frac{\partial P}{\partial y} \right)^2 \right]^{1/2}$



GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES

Equation (6.2) shows the requirement for the forces balance is the equalization between the Coriolis force and horizontal pressure. Thus, the horizontal flow vector is parallel to the broken isobaric lines in a direction that the larger isobaric lines in the Northern Hemisphere have located on the right in the direction of the flow and vice versa in the Southern Hemisphere. This type of flow is called the ***barotropic flow*** and the balance of forces represented by equation (6.2) is called the ***barotropic flow*** equilibrium.



GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES IN FLUIDS

Replace the horizontal pressure gradient in Equation (6.1) with the angle of inclination of the isobaric surfaces. Figure 6.1a shows the inclination of the isostatic surfaces relative to the equipotential surfaces. The nOz plane is perpendicular to the flow rate c . Pressure at point A is equal to “P” and at point B is equal to “ $P + \Delta P = P + \rho g \Delta z$ ”, where ρ – water column density between points C and B

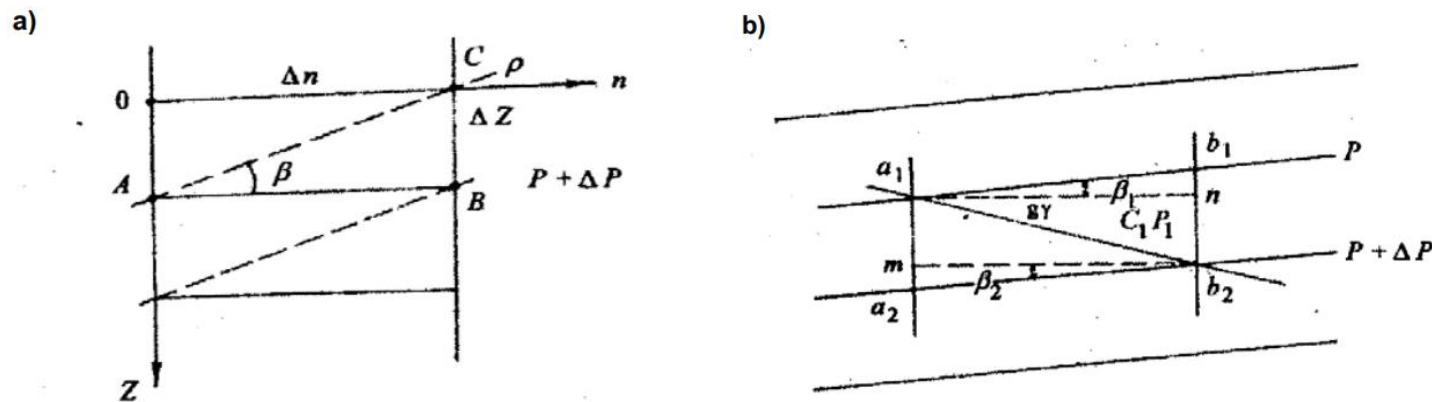


Figure 6.1. Inclination of isobaric surfaces



GEOSPHERE CURRENTS

❖ Inclination OF ISOBARIC SURFACES

Thus:

$$\begin{aligned}\frac{\Delta P}{\Delta n} &= \rho g \frac{\Delta z}{\Delta n} \\ \frac{\partial P}{\partial n} &= \rho g t g \beta\end{aligned}\quad (6.3)$$

If the Oz axis trend downwards, the angle β will follow clockwise rotation. From equations (6.2) and (6.3), the tg values β express as:

$$\text{tg } \beta = \frac{2\omega c \sin \varphi}{g}$$

From those equations, the angle of inclination of an isobaric surface is proportional to the flow rate at the depth of that surface.



GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES

The impact of the Coriolis force in actual currents creates the horizontal Circulation trend. As a result, lighter water in the upper layer moves on the right side of the flow direction and vice versa while the Southern Hemisphere sees the opposite movement. Therefore, with the same inclination of isobaric surfaces, the inclination of isopycnic surfaces appears. The angle between isobaric surfaces and isobaric volume is contrary. Moreover, the inclination of isopycnic surfaces appears somehow leads to the inclination of isobaric surfaces which causes horizontal gradient pressure. The distribution of isopycnic surfaces can relate to the water movement,



GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES IN FLUIDS

Figure 6.1b shows the inclination of the isopycnic curves relative to the isostatic surfaces. Since the pressures at points a2 and b2 are equal, then: :

$$g\rho_1(b_1b_2) = g\rho_2(a_1a_2) \quad (6.5)$$

Besides,

$$b_1b_2 = b_1n + b_2n$$

$$a_1a_2 = a_1m + a_2m$$

$$\frac{b_1n}{a_1n} = \operatorname{tg} \beta_1$$

$$\frac{a_2m}{b_2m} = \operatorname{tg} \beta_2$$

$$\frac{b_2n}{a_1n} = \operatorname{tg} \gamma$$

Ta change of expression (6.5), simplified g : $\rho_1(a_1n \operatorname{tg} \beta_1 + a_1n \operatorname{tg} \gamma) = \rho_2(a_1n \operatorname{tg} \gamma + a_1n \operatorname{tg} \beta_2)$ (6.6)

simplified the number of term a_1n and substitute (6.6) the values: $\operatorname{tg} \beta_1 = \frac{2\omega c_1 \sin \varphi}{g}$ và $\operatorname{tg} \beta_2 = \frac{2\omega c_2 \sin \varphi}{g}$

$$\operatorname{tg} \gamma = \frac{2\omega \sin \varphi}{g} \cdot \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_2 - \rho_1}$$

(6.7)



GEOSPHERE CURRENTS

❖ INCLINATION OF ISOBARIC SURFACES IN FLUIDS

In meteorology, this is similar to the Margules formula which is:

- 1) The position of the isopycnic lines on the cross-section allows consideration of the presence of flow perpendicular to the plane of the section and its direction;
- 2) The greater the inclination of the isopycnic lines, the smaller the density difference of the layers, the larger the speed difference. In immobilized layers, the isopycnic curves as well as the isobaric lines are horizontal
- 3) If layers of the same density move at different velocities, then $\gamma = 90^\circ$. In this case, the layers are very unstable and the γ has no meaning.



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

From the equations (6.1), find equation for u and v:

$$\begin{aligned} u &= -\frac{1}{2\omega\rho\sin\varphi} \cdot \frac{\partial P}{\partial y} = -\frac{\alpha}{2\omega\sin\varphi} \cdot \frac{\partial P}{\partial y} \\ v &= \frac{1}{2\omega\rho\sin\varphi} \cdot \frac{\partial P}{\partial x} = \frac{\alpha}{2\omega\sin\varphi} \cdot \frac{\partial P}{\partial x} \end{aligned} \quad (6.8)$$

These equations represent the balance between the horizontal composition of the friction force and the Coriolis force produced by the movement itself.

Because: $\alpha\partial P = \partial D$, then the expressions (6.8) can be rewritten as follows: $U = \frac{1}{2\omega\sin\varphi} \cdot \frac{\partial D}{\partial y}$ $V = \frac{1}{2\omega\sin\varphi} \cdot \frac{\partial D}{\partial x}$ (6.9)

If the direction of n is the greatest inclination of the isotropic side, then we have the speed:

$$c = \frac{1}{2\omega\sin\varphi} \cdot \frac{\partial D}{\partial n} \quad (6.10) \quad \text{Where } \frac{\partial D}{\partial n} \text{ actual inclination of the isobaric surfaces relative to the equipotential plane}$$

GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

The relative "inclination" between two hydrological stations is not difficult to determine. Suppose we have two hydrological stations A and B . Consider two isostatic surfaces P_1 and P_2 . Let the distance between A and B be ℓ . Then, for the isobaric surface P_1 , the flow rate in the direction perpendicular to AB is :

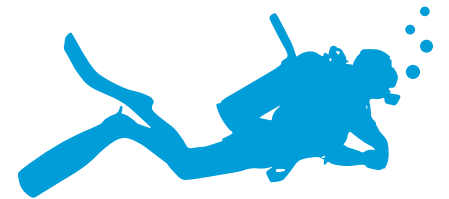
$$C_1 = \frac{D_{B_1} - D_{A_1}}{2\omega\ell \sin \varphi}$$

The line segment $AB = \ell$ is accepted as the differential factor dn , dynamic height D_{A_1} and D_{B_1} of the isobaric surface P_1 relative to the equipotential surface is unknown at present. The speed at the isobaric surface P_2 is determined by the same formula:

$$C_2 = \frac{D_{B_2} - D_{A_2}}{2\omega\ell \sin \varphi}$$

Take the first expression minus the second, we have results as below

$$C_1 - C_2 = \frac{(D_{B_1} - D_{B_2}) - (D_{A_1} - D_{A_2})}{2\omega\ell \sin \varphi} = \frac{\Delta D_B - \Delta D_A}{2\omega\ell \sin \varphi} \quad (6.11)$$



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

Thus, the dynamic method only allows for determining the difference in speeds. If we know the flow rate at a certain cross-section (where the flow velocity is zero), the problem is simply solved. But in practice, we almost always do not know that speed, so the problem is how to choose the zero surface. Thanks to equation (6.11), it is possible to calculate the real speed of the flow at different water levels.

Based on equation (6.10), it can be determined that the zero surface is the depth at which the horizontal gradient components of the dynamic depth approach 0.



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

The Defant method is the most common method for choosing the zero side, which is based entirely on the dynamic characteristics of the flow and does not contain assumptions like other methods.

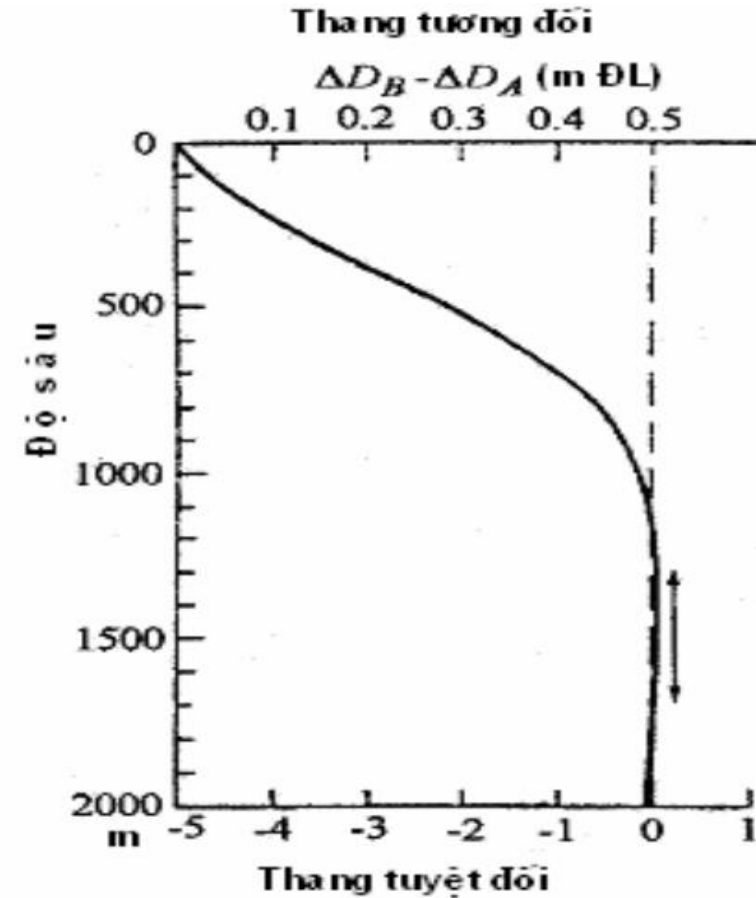


Figure 6.2. To identify the face without the Defant method

GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

When searching for zero surfaces, Defant noticed that most of the differences in curves between dynamic depths in two oceanographic stations (Figure 6.2) for different station pairs are characterized by the existence of more or less straight segments. standing, for pairs of neighboring stations, they are distributed at approximately the same depths.

Within those segments, the dynamic depth differences remain constant. This means the speeds of the currents are identical.

If the 0 mark is not located in the vicinity of this vertical segment, then in the whole layers that have no difference in flow rates. If the 0 mark is not located far enough from this section, then the flow rates in the entire layer will be equally large. The latter is also less realistic, so Defant assumes that the flow rates throughout the layer are equal to the difference between the same dynamic depths, while the zero surface is in the center of the layer.



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

There is another method is use to select the zero surface based on the analysis of the individual volume difference curves between neighbouring stations – Parr's method. This method is attributed to the identification of variations in water layers between selected mass isomorphs and so on.

Since the fact that in the World Ocean does not exist on a single (continuous) side, instead of using a reference side as common, the geosphere currents are assumed to equal zero. For this purpose, the reference surface chosen in the class between 1000 and 2000 m is quite appropriate, while the surface at 3000 m of depth is chosen in some examples such as the southern Ocean



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

Other difficulties in determining the zero side, the method of dynamic calculation also has a series of disadvantages:

- As not mention to the pure flow component under the direct impact of tangential wind stress, exclusive the speed and direction of the wind, the swirling and non-stop components caused by forces not present in the basic equation (6.11), as well as ignoring the seabed topography.
- In addition, large deviations in flow rate can appear if the hydrological cross section is stimulated for a long period of time, furthermore not perpendicular to the direction of the flow, the distance between stations is not the same and quite large, especially in areas with front.



GEOSPHERE CURRENTS

❖ METHOD OF DYNAMIC CALCULATION OF GEOSPHERE CURRENTS

Despite such major drawbacks, the method of dynamic calculation due to simplicity and ease of use has been recognized worldwide and remains valid to this day. This method is often applied to standard cross sections, when performing standard cross sections always have to compare the results received with the estimated data in previous years. We also note that ocean circulation maps built on dynamic methods (Shott, 1933, Sverdrup, 1941, Ditrach, 1961, etc.) are generally quite consistent with observational data and overall ocean circulation mathematical modeling results.



GEOSPHERE CURRENTS

❖ DYNAMIC METHODS FOR CALCULATION OF GEOGRAPHIC FLUID

Figure 6.3 shows the dynamic surface map of Nam Duong as an example.

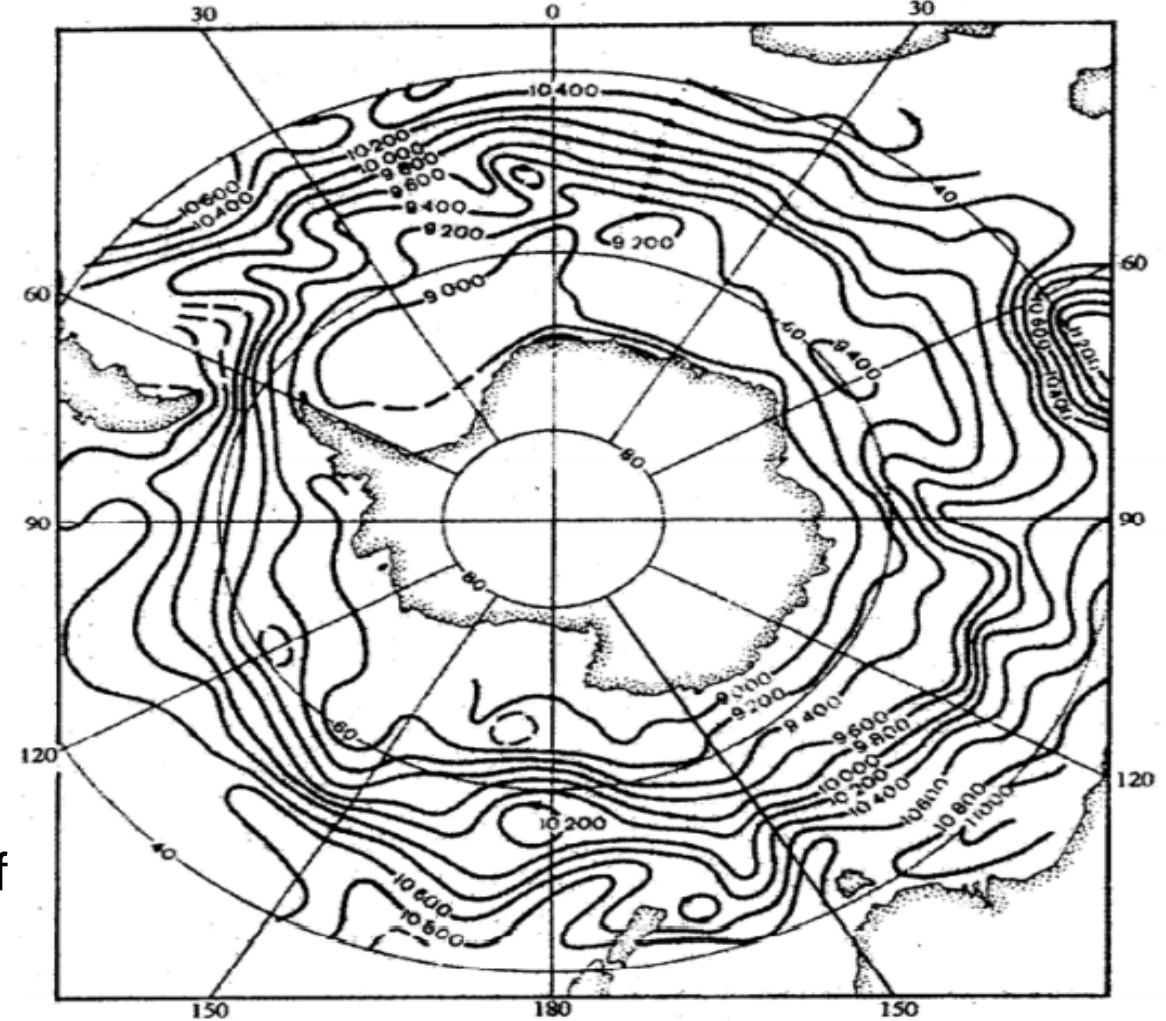


Figure 6.3: The dynamic surface map of Nam Duong as an example

THEORY OF CURRENTS



STEADY CURRENTS

❖ Ekman theory with deep currents

Since the friction stress of the wind is greater than the other forces that cause the flow, on average the wind flow contributes the largest part to the total speed of the flows, especially in the upper layer of the ocean. Ekman made the following assumptions:

1. The sea is shoreless and infinitely deep (to eliminate the effect of friction with the shore and bottom);
2. Wind and currents caused by it are stable and do not change over time;
3. The wind and current speed fields do not vary in the horizontal direction (no divergence);
4. The vertical component of the speed is absent because the motion occurs only in the horizontal direction and does not diverge;
5. Sea is homogeneous in density (to exclude density flow) and incompressible water;
6. The sea surface is the horizontal plane (to exclude the gradient component);
7. The accepted coefficient of tangential friction A_z remains constant with depth



STEADY CURRENTS

❖ Ekman theory with deep currents

With all the assumptions for steady flow, a turbulent frictional is the only force that transmits the impact of wind stress down to the depth and the Coriolis force is equal to it. The equation of motion in this case has the form:

$$\frac{A_z}{\rho} \frac{d^2 u}{dz^2} + 2\omega v \sin \varphi = 0$$

$$\frac{A_z}{\rho} \frac{d^2 v}{dz^2} - 2\omega u \sin \varphi = 0$$

Here we put:

- The Y axis to coincide with the wind direction,
- The X axis is towards the right,
- The Z axis is pointing down

Transform the expressions above into forms:

$$\frac{d^2 u}{dz^2} + \frac{2\rho}{A_z} \omega v \sin \varphi = 0 \quad (6.12)$$

$$\frac{d^2 v}{dz^2} - \frac{2\rho}{A_z} \omega u \sin \varphi = 0$$

If symbolize: $\frac{\rho \omega \sin \varphi}{A_z} = a^2$

Then the equations (6.12) are rewritten to:

$$\frac{d^2 u}{dz^2} + 2a^2 v = 0 \quad (6.13)$$

$$\frac{d^2 v}{dz^2} - 2a^2 u = 0$$



❖ Ekman theory with deep currents

This is a system of second-order ordinary differential equations and solutions of the form :

$$\begin{aligned} u &= c_1 e^{az} \cos(az + \phi_1) + c_2 e^{-az} \cos(az + \phi_2), \\ v &= c_1 e^{az} \sin(az + \phi_1) - c_2 e^{-az} \sin(az + \phi_2), \end{aligned} \quad (c_1, c_2, \phi_1, \phi_2 - \text{constants})$$

We state the first boundary condition: the flow rate when increasing depth needs to be limited, i.e.

$$u \neq \infty, \quad v \neq \infty \quad \text{when} \quad z \rightarrow \infty$$

In this case, c_1 must be zero, otherwise, the increasing of (z) the speed will increase infinitely. At the same time, it is no longer necessary to identify ϕ_1 .

We rewrite the equations (6.14) as follows: $u = c_2 e^{-az} \cos(az + \phi_2); \quad v = -c_2 e^{-az} \sin(az + \phi_2)$ (6.15)

We set out the second boundary condition:

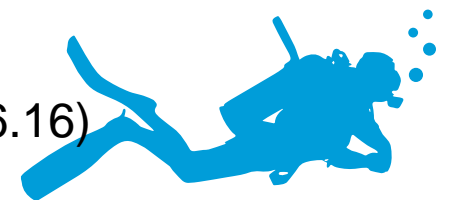
- At sea surface $z = 0$
- Wind tangential stress
- And axis Y in the direction θ .

$$\tau = A_z \frac{dc}{dz}$$

Then, at $z = 0$ and the edge stress in the water just below the ocean surface will be equal to the wind gland friction, we have:

$$\begin{aligned} -A_z \frac{du}{dz} &= 0 \\ -A_z \frac{dv}{dz} &= \tau \end{aligned}$$

(6.16)



STEADY CURRENTS

❖ Ekman theory with deep currents

The speed module symbol at the surface is U_0 , when:

$$U_0 = \sqrt{u^2 + v^2} = \frac{\tau}{\sqrt{2}A_z a}. \quad (6.17)$$

Substituting the value to equation (6.17), we get

$$U_0 = \sqrt{u^2 + v^2} = \frac{\tau}{\sqrt{2}A_z a}. \quad (6.18)$$

Equation (6.17) can draw the conclusion that with the same conditions, the flow velocity decreases as latitude increases. Along with (6.17) the equations (6.15) can be rewritten

$$\begin{aligned} u &= U_0 e^{-az} \cos(45 - az) \\ v &= U_0 e^{-az} \sin(45 - az) \end{aligned} \quad (6.19)$$



STEADY CURRENTS

❖ Ekman's theory with deep current

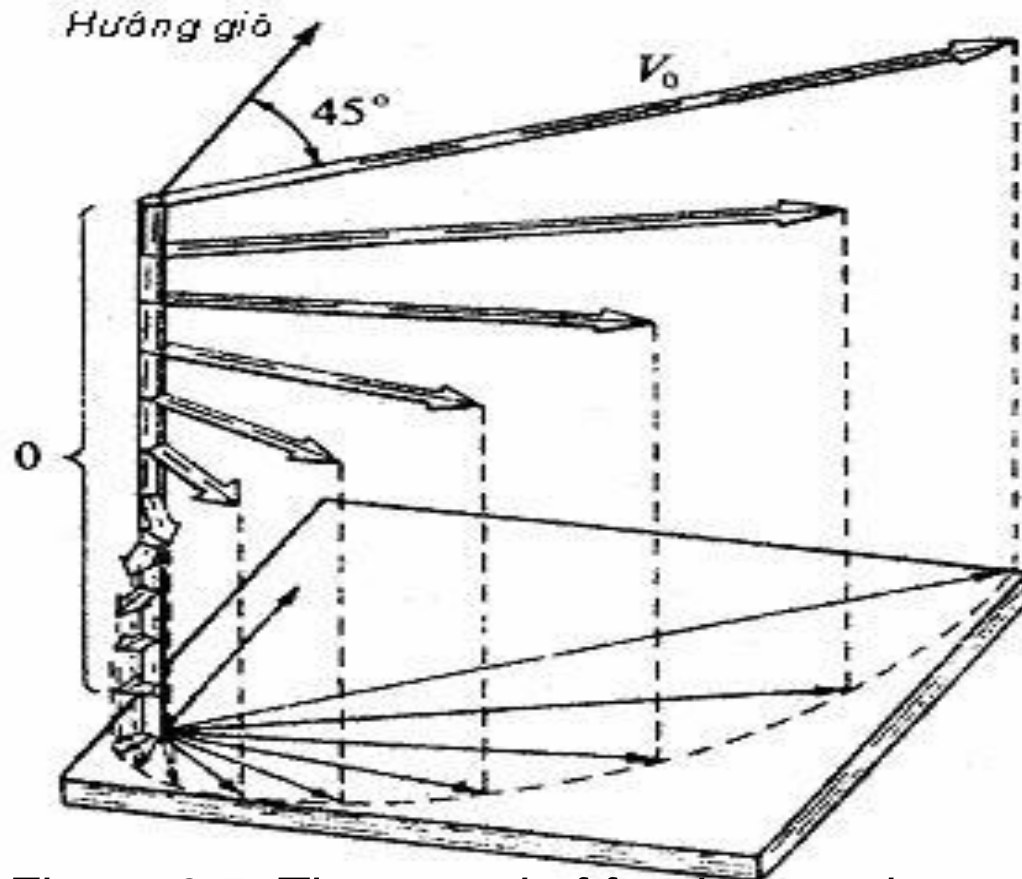


Figure 6.5. The speed of fundamental currents
according to Ekman

Along with a decrease in speed with depth, the current turns to the right relative to its direction at sea level.

Figure 6.4 represents the speed line described in the shape of loga twist and represents a change in direction and the speed of wind flow in depth. Figure 6.4 shows that, at some depth, the speed vector will point in the opposite direction to the face flow.

It is often referred to as the depth of friction (rather the impact depth of friction) and the symbol in D.:

$$D = \frac{\pi}{a} = \pi \sqrt{\frac{A_z}{\rho \omega \sin \varphi}}$$

(6.20)



STEADY CURRENTS

❖ Ekman theory with deep currents

The quantity A_z is difficult to determine, so when there is flow monitoring data in the ocean surface layer, it is possible to find A_z from formula (6.20) if the quantity D is known:

$$A_z = \frac{D^2 \rho \omega \sin \varphi}{\pi^2} \quad (6.21)$$

The total flux of the drift is determined by integrating from zero to infinity in the coordinate axes directions:

$$S_x = \int_0^\infty u dz \quad \text{v} \hat{a} \quad S_y = \int_0^\infty v dz \quad (6.22)$$

Substitute u and v from (6.19) to (6.22):

$$S_x = U_0 \int_0^\infty e^{-az} \cos(45^\circ - az) dz \quad S_y = U_0 \int_0^\infty e^{-az} \sin(45^\circ - az) dz$$



❖ Ekman's theory for the deep sea

$$\int e^{az} \sin bx \, dx = \frac{e^{az}}{a^2 + b^2} (a \sin bx - b \cos bx) \quad , \quad \int e^{az} \cos bx \, dx = \frac{e^{az}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$S_x = \frac{U_0 e^{-az}}{2a^2} [-a \cos(45 - az) - a \sin(45 - az)] \Big|_0^\infty = \frac{U_0}{2a^2} a \sqrt{2} = \frac{U_0 \sqrt{2}}{2a}$$

$$= U_0 \sqrt{2} / 2a \cdot \pi / a \cdot a / \pi = \frac{U_0 \sqrt{2} D}{2\pi}$$

$$S_y = \frac{U_0 e^{-az}}{2a^2} [-a \sin(45 - az) + a \cos(45 - az)] \Big|_0^\infty = 0$$

$$S_x = \frac{U_0 \sqrt{2} D}{2\pi}, \quad S_y = 0 \quad (6.23)$$



STEADY CURRENTS

❖ Ekman's theory for the shallow sea

There is no difference in results for the shallow sea. By integral equation (6.13) and sets the additional conditions so that at the seabed both the speed components of u and v are equal to zero.

Without repeating all of Ekman's arguments, we write:

$$\begin{aligned} u &= A \sinh a \xi \cos a \xi - B \cosh a \xi \sin a \xi \\ v &= A \cosh a \xi \sin a \xi + B \sinh a \xi \cos a \xi \end{aligned}$$

ξ - vertical coordinates at the base.

The constants analyzed by A and B are equal to:

$$\begin{aligned} A &= \frac{\tau D}{\pi A_z} \frac{\cosh a d \cos a d + \sinh a d \sin a d}{\cosh^2 a d + \cos^2 a d} \\ B &= \frac{\tau D}{\pi A_z} \frac{\cosh a d \cos a d - \sinh a d \sin a d}{\cosh^2 a d + \cos^2 a d} \end{aligned} \quad (6.24)$$

With d - is sea depth



STEADY CURRENTS

❖ Ekman's theory for the shallow sea

The angle between the flow direction at the surface and axis Y is determined by the expression:

$$\tan(U_0, Y) = \frac{U_0}{V_0} = \frac{\sinh 2ad - \sin 2ad}{\sinh 2ad + \sin 2ad} \quad (6.25) \quad \text{where } 2ad \text{ is the depth of sea } 2ad = 2ad \frac{\pi a}{a \pi} = \frac{2\pi d}{D}$$

Then the quantity d/D can be considered as shallow water indicator

The table below shows the α value between the flow vector and the wind vector that depends on the quantity d/D

d/D	0.1	0.25	0.5	0.75	1	>1
α	5	21.5	45	45.5	45	45

From Figure 6.5 this infers that, when $d > D$ the speed of the vectors the actual flow speed coincides with the case of the infinite deep sea (see Figure 6.4).

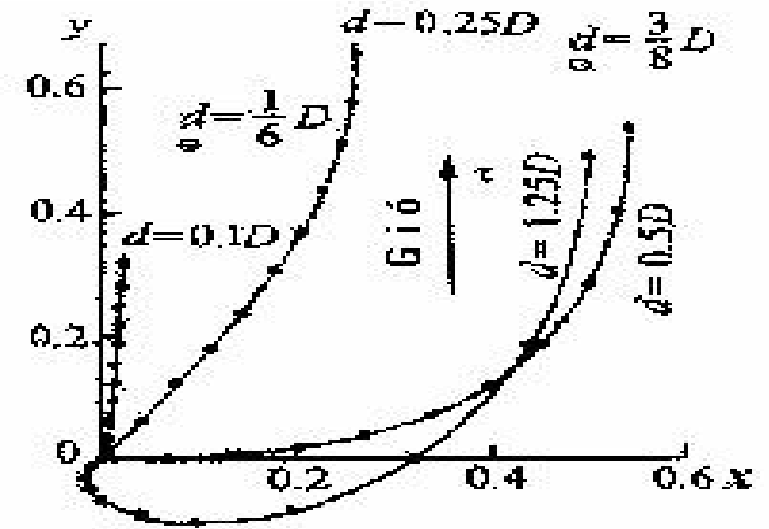


Figure 6.5 Current velocity heads in the sea of finite depth depending on the d/D ratio



STEADY CURRENTS

❖ The development of fluency currents

Before stabilizing, the direction and velocity of flow may be much different than defined by formulas (6.19) and (6.24). Ekman looked at the development of drift currents in case the wind with constant intensity and direction began to effect the silent sea surface in a stable state. There is seeing that the flow at different water levels develops differently and the deeper you dive, the later stability appears.



STEADY CURRENTS

❖ The development of flowing currents

In Figure 6.6, the endpoint of the unstable flow vector draws a complex spiral-shaped curve that gradually approaches a stable value.

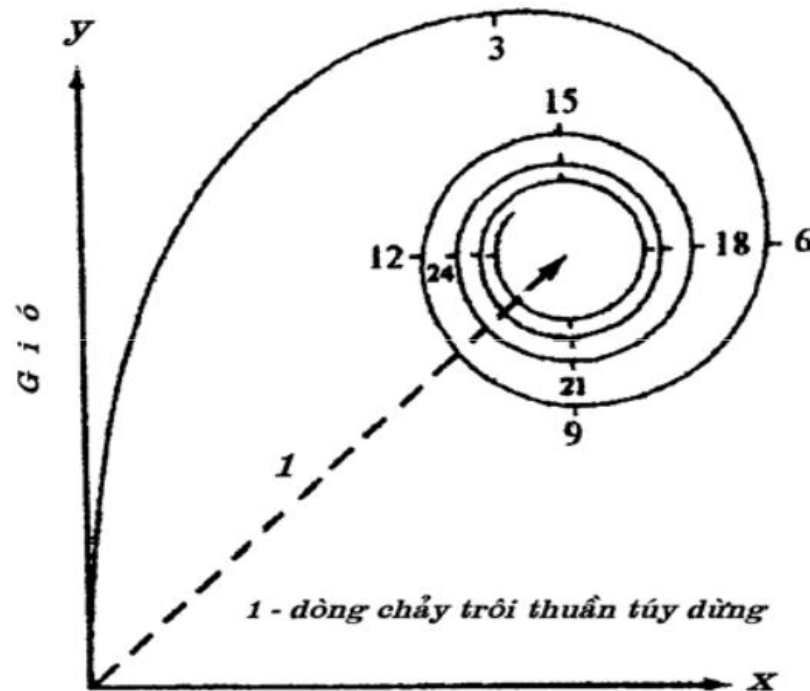


Figure 6.6. The velocity curve shows the development of pure drift current at the sea surface (time from wind arrival is constant equal to pendulum hours).



GRADIENT CURRENTS THEORY

In nature, the rise and fall of water surface occurs even far from shore. The inclination of the sea surface can create a pressure gradient that causes the gradient currents. Ekman made the following assumptions to simplify the process:

- 1) The landless sea and homogeneous in density;
- 2) The inclination of the sea surface is constant and stable in time and space;
- 3) Flat seabed;
- 4) Stable flow, no vertical components;
- 5) There is no fluctuation in the turbulent viscosity coefficient with depth.



GRADIENT CURRENTS THEORY

In this case, the following impact forces are the horizontal pressure, the Coriolis force, and the friction force in which the bottom friction is transmitted vertically, which constrains movement.

Motion equations write in the form of:

$$\frac{A_z}{\rho} \frac{d^2 u}{dz^2} + 2\omega v \sin \varphi = 0 \quad \frac{A_z}{\rho} \frac{d^2 v}{dz^2} - 2\omega u \sin \varphi + g \sin \beta = 0 \quad (6.26)$$

Formulas for determining gradient flow rate components are written as:

$$u = \frac{g \sin \beta}{2\omega \sin \varphi} \left[1 - \frac{\operatorname{ch} a(H+z) \cos a(H-z) + \operatorname{ch} a(H-z) \cos(H+z)}{\operatorname{ch} 2aH + \cos 2aH} \right] \quad (6.27)$$

$$v = \frac{g \sin \beta}{2\omega \sin \varphi} \left[\frac{\operatorname{sh} a(H+z) \sin a(H-z) + \operatorname{sh} a(H-z) \sin(H+z)}{\operatorname{ch} 2aH + \cos 2aH} \right]$$



GRADIENT CURRENTS THEORY

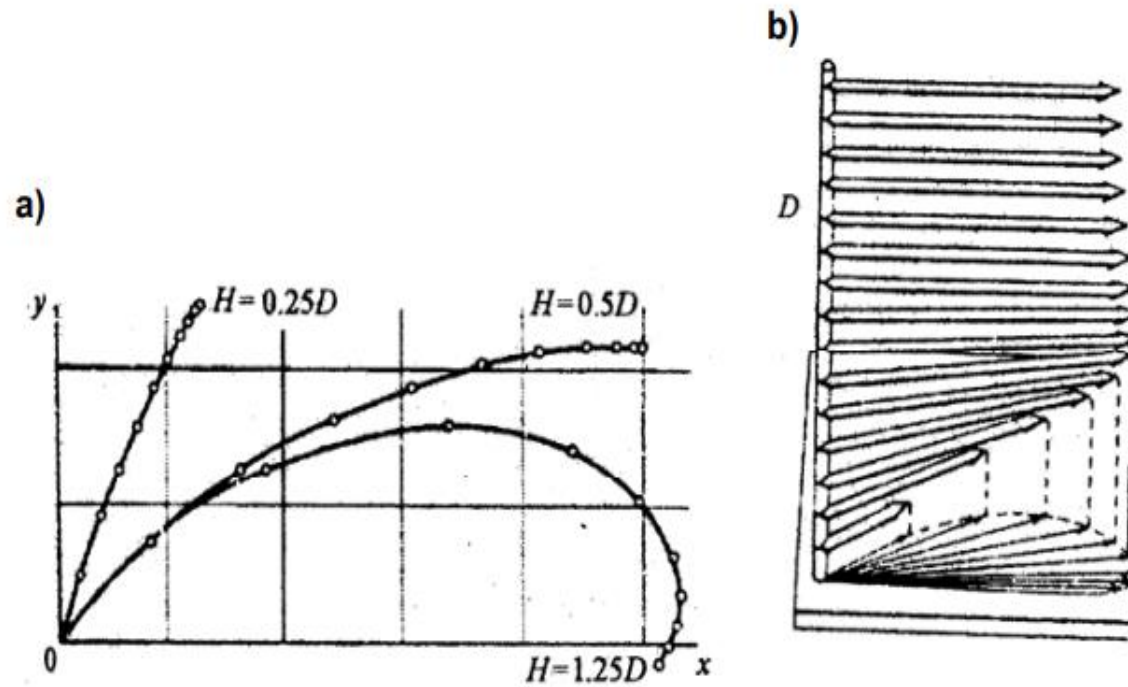


Figure 6.7.

Based on equations (6.27), (Figure 6.7a), the construction of the first curve's velocity current for three values of sea depth is expressed as a fraction of the depth friction. Figure 6.7 shows the stereoscopic change of gradient flow in different depths.



GRADIENT CURRENTS THEORY

At the seabed, the currents are zero according to the condition. As the increase in distance from the bottom, the flow velocity increases slowly turns to the right direction compared to the inclination of the water level. As the water depth large enough, maximum speed and deflection angle 90° achieved at a distance $D = \frac{\pi}{a}$ from the bottom. Since continuing leave from the bottom, the speed and direction of the flow remain constant until reaching the surface.

As such, the influence of bottom friction is spread upwards within the range layer of the D-thickness. Similar to the influence of the depth of friction in the Ekman drift, this layer is called the lower friction (the lower boundary of the depth influence the bottom friction).

The total gradient flux has components in both coordinate axes. The Y-axis composition is only significant in the layers near the bottom and when $H > D$ it reaches a defined finite limit like the X-axis component:

$$S_y \rightarrow \frac{D g \sin \beta}{4\pi \omega \sin \varphi}, S_x \rightarrow \frac{g \sin \beta}{2\omega \sin \varphi} \left(H - \frac{D}{2\pi} \right)$$

(6.28)

