LECTURE

MODELLING THE MARINE ENVIRONMENT

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Lecture 9

STRETER- PHELPS MODEL

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- Point source
- Distribution source

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STRETER- PHELPS POINT SOURCES

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Mass balance equation:

BOD: $$V\frac{dL}{dt} = -k_d VL$$

DO: $$V\frac{do}{dt} = -k_d VL + k_a V(o_s - o)$$

DO Deficit: $D = o_s - o$ $\longrightarrow \frac{dD}{dt} = -\frac{do}{dt}$

$$\longrightarrow \quad V\frac{dD}{dt} = k_d VL - k_a VD$$

If at = 0: L = L_o and D = O, above equation can be differentiated

$$L = L_0 e^{-k_d t}$$

$$D = \frac{k_d L_0}{k_a - k_d}\left(e^{-k_d t} - e^{-k_a t}\right)$$

EXPERIMENT (Cont)

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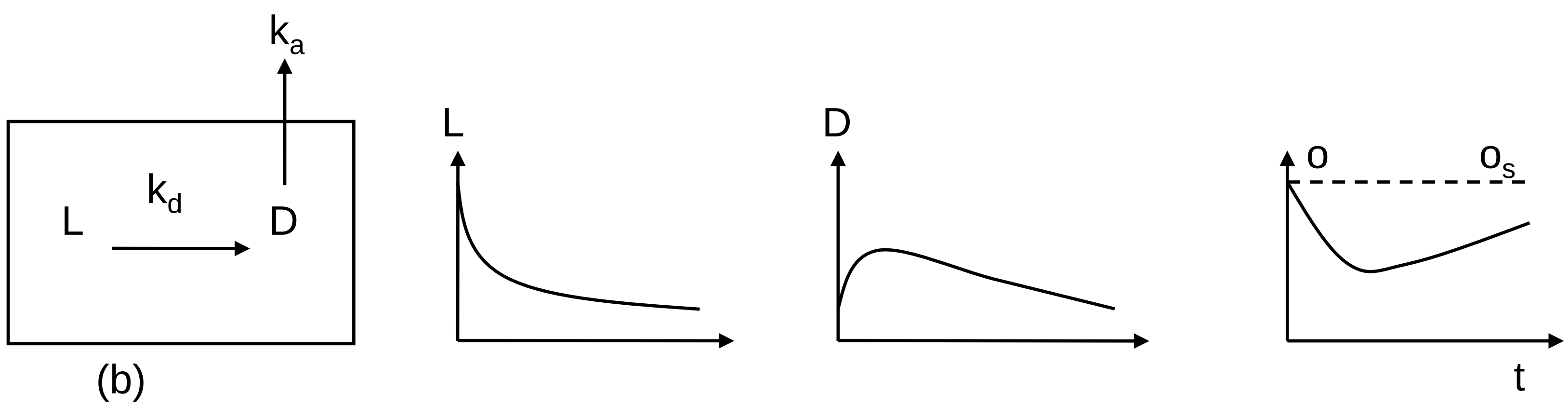

BOD decomposition process in (a) closed and (b) open system

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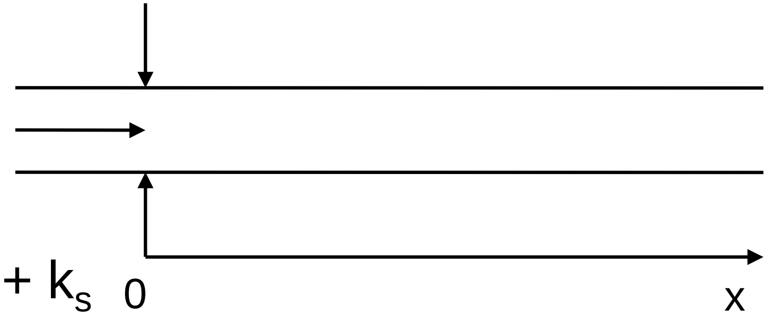
POINTS SOURCE -STREETER-PHELPS EQUATION

A stream with a single-point source of BOD. The river segment is in a steady state and is characterized by plug flow with constant hydrological and geometry (PFR system). This is the simplest manifestation of the classic Streeter – Pheld model

Mass balance: $0 = -U\frac{dL}{dx} - k_r L$



$$0 = -U\frac{dD}{dx} + k_d L - k_a D$$

với $k_r = k_d + k_s$

If $L = L_o$ và $D = D_o$ at t = 0, then these equation can be solved for

$$L = L_0 e^{-\frac{k_r}{U}x}$$

$$D = D_0 e^{-\frac{k_a}{U}x} + \frac{k_d L_0}{k_a - k_r}\left(e^{-\frac{k_r}{U}x} - e^{-\frac{k_a}{U}x}\right)$$

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POINTS SOURCE -STREETER-PHELPS EQUATION (cont)

❖ **Some formulas calculate reaeration coefficient (k_a)**

O'Connor-Dobbins (1956)

$$k_a = 3.93\frac{U^{0.5}}{H^{1.5}}$$

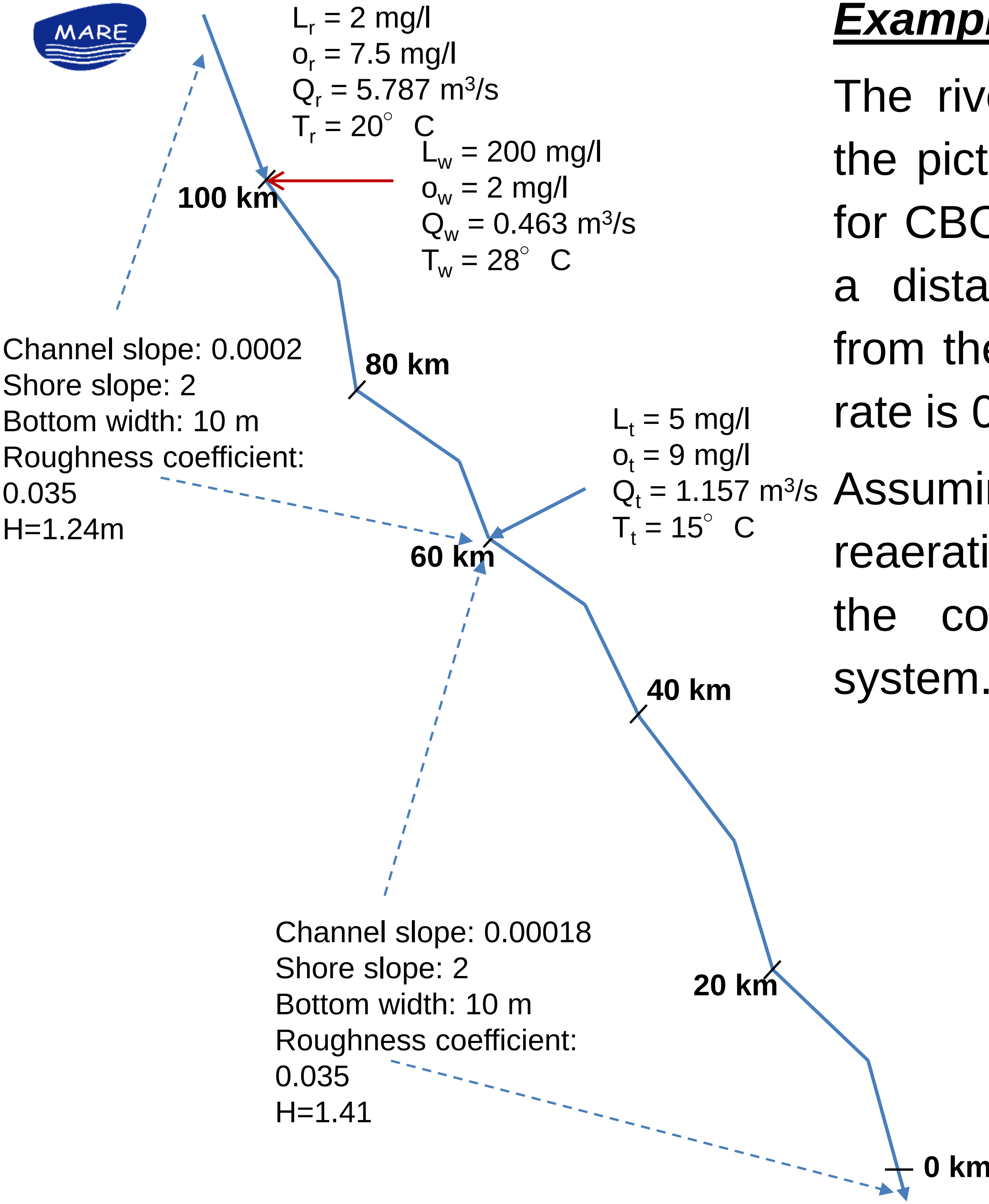
Churchill (1962)

$$k_a = 5.026\frac{U}{H^{1.67}}$$

Owens và Gibbs (1964)

$$k_a = 5.32\frac{U^{0.67}}{H^{1.85}}$$

where: k_a (1/day), U (m/s), H (m)



Example 1:

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The river is described as shown in the picture. The de-oxygenation rate for CBOD is 0.5 day^{-1} in 20^{o}C. From a distance of 20 km downstream from the plant, the CBOD deposition rate is 0.25 day^{-1}.

Assuming that O'Connor-Dobbins's reaeration formula is used, calculates the concentration of DO in the system.

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The hydrographic morphological parameters and reaction kinetics of the system

Parameter	Units	KP > 100	KP: 100-60	KP < 60
Depth	m	1.19	1.24	1.41
Area	m^2	14.71	15.5	18.05
Flow	$m^3\ s^{-1}$	5.787	6.250	7.407
	$m^3\ d^{-1}$	500,000	540,000	640,000
Velocity	$m\ s^{-1}$	0.393	0.403	0.410
	$m\ d^{-1}$	33,955	34,891	35,524

Parameter	KP > 100	KP: 100 - 80	KP: 80 - 60	KP < 60
T (^oC)	20	20.59	20.59	19.72
O_s (mgL^{-1})	9.092	8.987	8.987	9.143
k_a (d^{-1})	1.902	1.842	1.842	1.494
k_r (d^{-1})	0.50	0.764	0.514	0.494
k_d (d^{-1})	0.50	0.514	0.514	0.494

ESTUARY STREETER-PHELPS MODEL

With a diffusion system such as an estuary the Streeter-Phelps equation can be written as:

$$0 = E\frac{d^2L}{dx^2} - U\frac{dL}{dx} - k_r L$$

$$0 = E\frac{d^2D}{dx^2} - U\frac{dD}{dx} + k_d L - k_a D$$

If L = L_o và D = D_o at t = 0, the solution for BOD is

$$L = L_0 e^{j_{1r}x} \qquad x \leq 0$$

$$L = L_0 e^{j_{2r}x} \qquad x \geq 0$$

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ESTUARY STREETER-PHELPS MODEL (cont)

The solution to oxygen deficiency

$$D = \frac{k_d}{k_a - k_r} \frac{W}{Q} \left(\frac{e^{j_{1r}x}}{\alpha_r} - \frac{e^{j_{1a}x}}{\alpha_a} \right) \qquad x \leq 0$$

$$D = \frac{k_d}{k_a - k_r} \frac{W}{Q} \left(\frac{e^{j_{2r}x}}{\alpha_r} - \frac{e^{j_{2a}x}}{\alpha_a} \right) \qquad x \geq 0$$

where

$$L_0 = \frac{W}{\alpha_r Q}$$

$$\alpha_r = \sqrt{1 + \frac{4k_r E}{U^2}}$$

$$\alpha_a = \sqrt{1 + \frac{4k_a E}{U^2}}$$

$$\begin{matrix} j_{1r} \\ j_{2r} \end{matrix} = \frac{U}{2E}\left(1 \pm \alpha_r\right)$$

$$\begin{matrix} j_{1a} \\ j_{2a} \end{matrix} = \frac{U}{2E}\left(1 \pm \alpha_a\right)$$

ESTUARY STREETER-PHELPS MODEL (cont)

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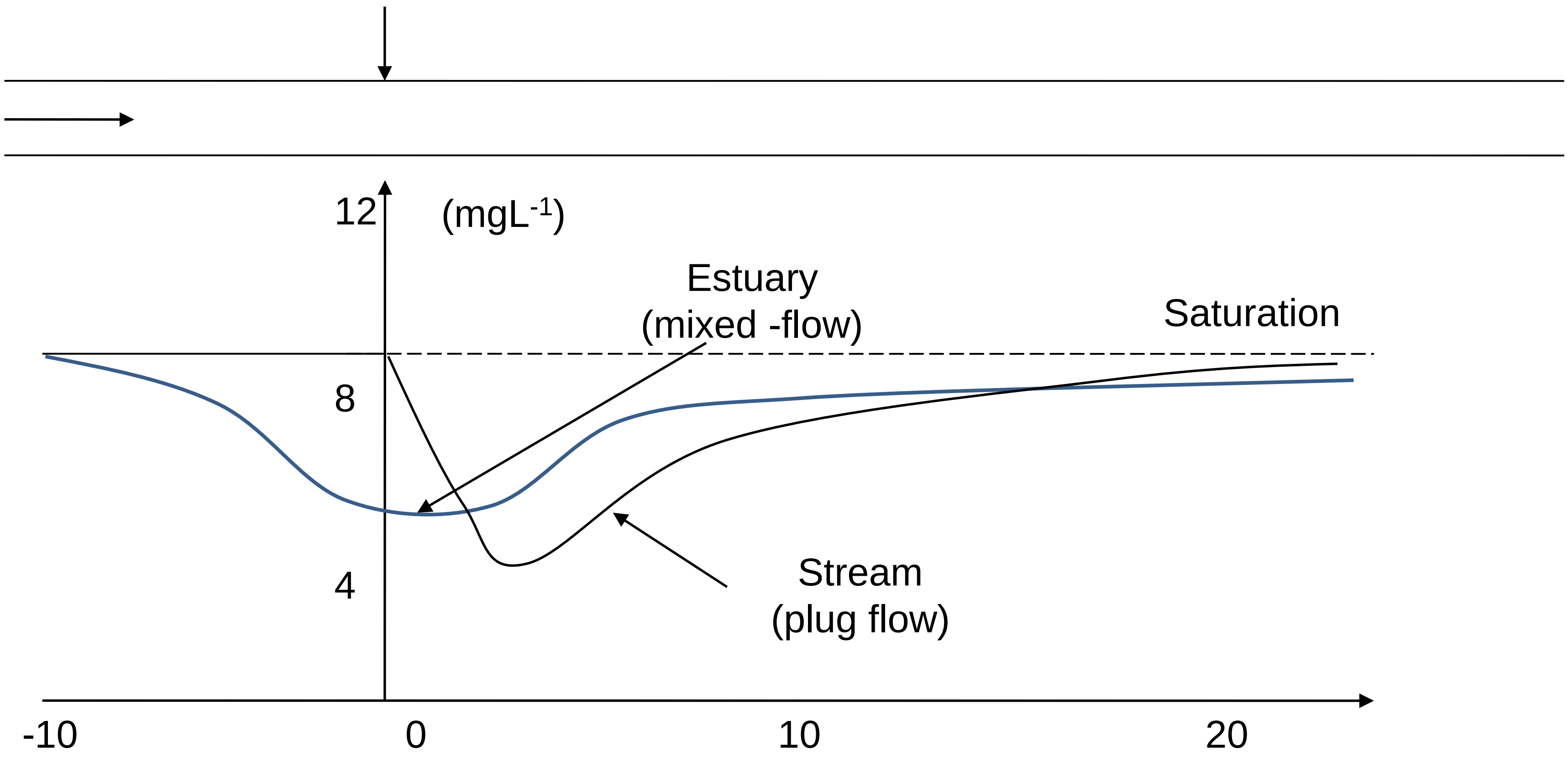

DO chart for a point waste source into an estuary

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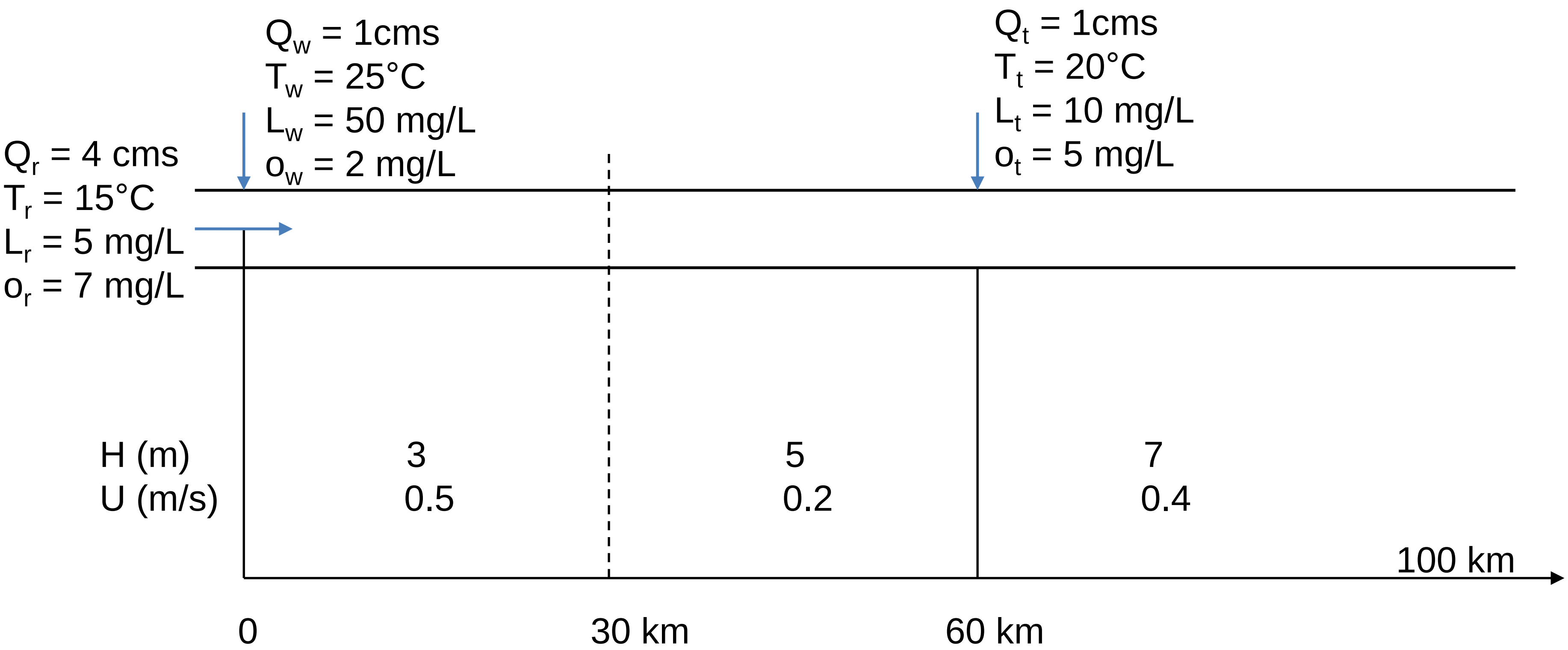
<u>Exercises 1:</u> Determine BOD and DO for the following section of the river.

For the decomposition rate of BOD at 20°C is 0.35 (1/day), θ = 1.047.



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STRETER- PHELPS DISSTRIBUTED SOURCES

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PARAMETERIZATION OF DISTRIBUTION SOURCES

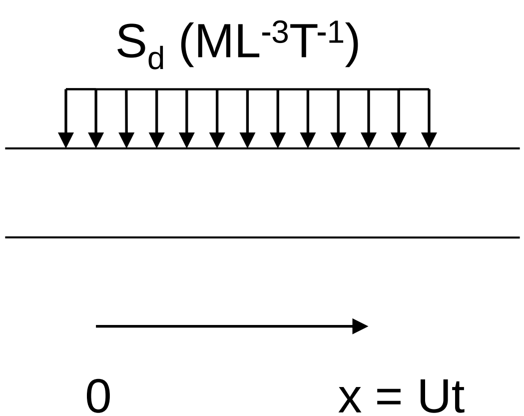
Mass balance in case of plug- flow: $$0 = -U\frac{dc}{dx} - kc + S_d$$

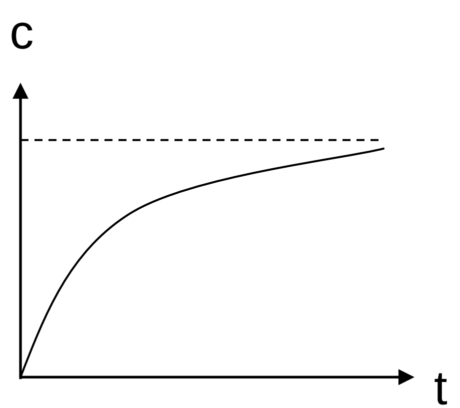
If at = 0: c = c_o , then the solution is:

$$c = \frac{S_d}{k}\left(1 - e^{-kt}\right)$$

S_d = the rate of the distribution source. ($ML^{-3}T^{-1}$).

t = travel time , t = x/U.





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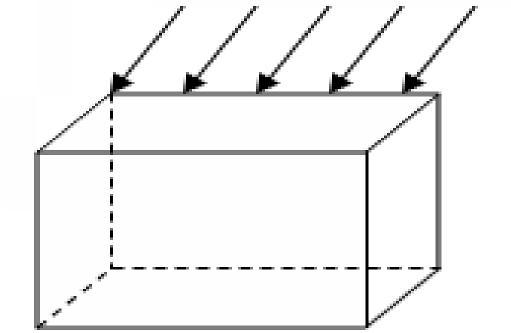
PARAMETERIZATION OF DISTRIBUTION SOURCES (cont)

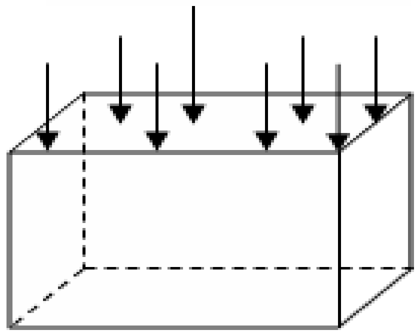
❖ Distributed source types

$S_d''\ (ML^{-1}T^{-1})$



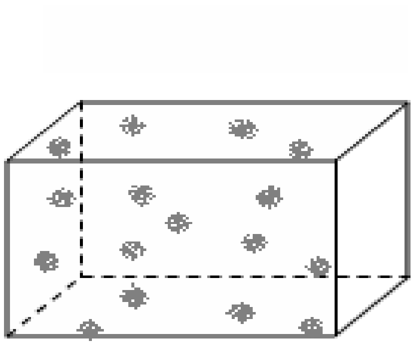
(a) Line load

$S_d'\ (ML^{-2}T^{-1})$



(b) Area load

$S_d\ (ML^{-3}T^{-1})$



(c) Volume load

$$S_d = S_d'' \frac{L}{V} = \frac{S_d''}{A_c}$$

$$S_d = S_d' \frac{A_s}{V} = \frac{S_d'}{H}$$

L = distribution source length

V = volume

A_c = the cross-section area of the segment in which the source is included.

BOD DISTRIBUTION SOURCE

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- Source: bottom sludge with high organic concentration

  Mass balance:

$$0 = -U\frac{dL}{dx} - k_r L + S_L$$

Where S_L = the speed of the BOD distribution source($gm^{-3}d^{-1}$)

k_r = rate of BOD removal (day^{-1}).

Solution when t = 0: L = 0:

$$L = \frac{S_L}{k_r}\left(1 - e^{-k_r t}\right)$$

BOD DISTRIBUTION SOURCE (cont)

The mass balance for oxygen deficiency is written as follows

$$0 = -U\frac{dD}{dx} - k_a D + \frac{k_d}{k_r} S_L \left(1 - e^{-k_r t}\right)$$

At t = 0: D = 0, The solution to mass balance is as follows:

$$D = \frac{k_d S_L}{k_r k_a}\left(1 - e^{-k_a t}\right) - \frac{k_d S_L}{k_r \left(k_a - k_r\right)}\left(e^{-k_r t} - e^{-k_a t}\right)$$

Where t = travel time. (t = x/U)

DO DISTRIBUTION SOURCE

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- Source: Plants (photosynthesis)
- Deposition: plants (respiration), SOD
- Mass balance

$$0 = -U\frac{dD}{dx} - k_a D - P + R + \frac{S_B^{'}}{H}$$

Where P, R: the rate of photosynthesis and respiration of plants ($gm^{-3}nday^{-1}$)

$S_B^{'}$ = oxygen demand rate for sediment ($gm^{-2}day^{-1}$)

H = depth (m)

- The solution when t = 0: L = 0:

$$D = \frac{-P + R + \left(S_B^{'}/H\right)}{k_a}\left(1 - e^{-k_a t}\right)$$

TOTAL STREETER-PHELPS MODEL

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- General solution to the points and distribution sources of BDO and DO.
- $$L = \boxed{L_0 e^{-k_r t}} + \boxed{\frac{S_L}{k_r}\left(1 - e^{-k_r t}\right)}$$

Point source

Distribution source

Point deficit

Point BOD

$$D = \boxed{D_0 e^{-k_r t}} + \boxed{\frac{k_d L_0}{k_a - k_r}\left(e^{-k_r t} - e^{-k_a t}\right)}$$

$$\boxed{+\frac{-P + R + \left(S_B^{'}/H\right)}{k_a}\left(1 - e^{-k_a t}\right)}$$

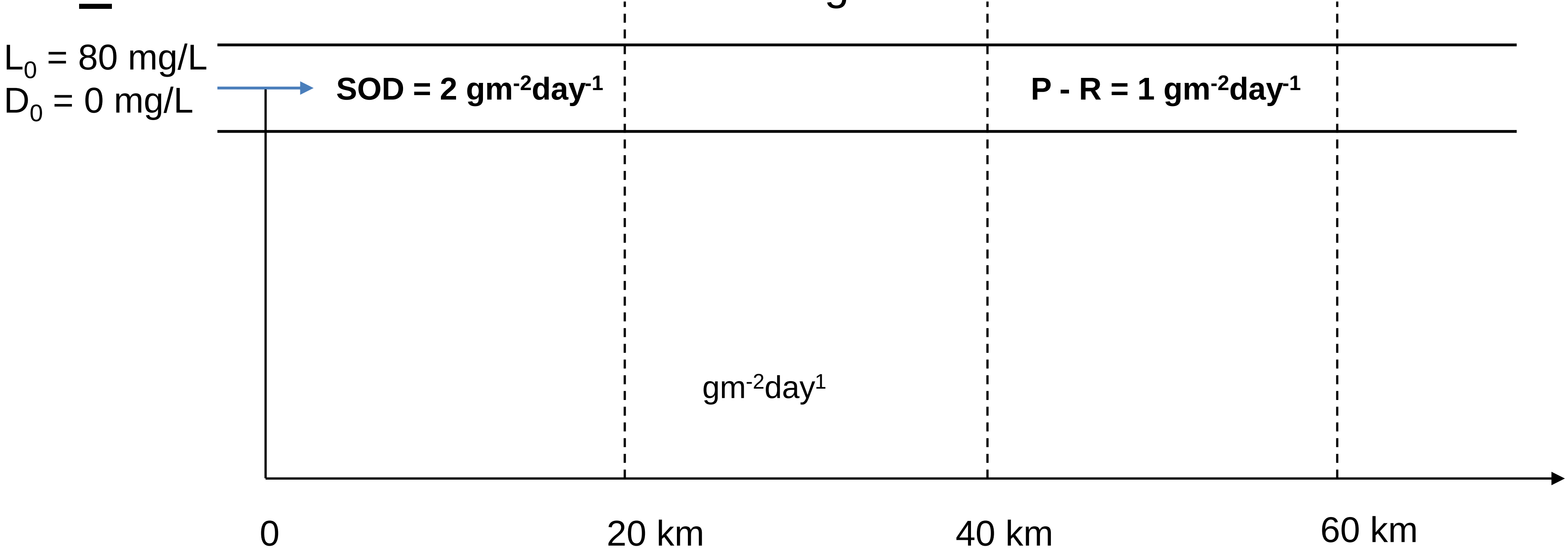
Distribution Deficit

$$\boxed{+\frac{k_d S_L}{k_r k_a}\left(1 - e^{-k_a t}\right) - \frac{k_d S_L}{k_r\left(k_a - k_r\right)}\left(e^{-k_r t} - e^{-k_a t}\right)}$$

Distributed BOD

Exercises:

Determine bod and do for the following section of the river.



U (mps)	0.1	0.15	0.1
H (m)	0.8	1	1
k_r (ngày^{-1})	0.2	0.1	0.1
k_d (ngày^{-1})	0.1	0.1	0.1
k_a (ngày^{-1})	1	1.2	1.2
o_s (mgL^{-1})	10	9	8