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TOPIC 2

WAVES

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COURSE LEARNING OUTCOMES

Upon completion of this course, students should be able to:

1. Evaluate the properties of offshore and near shore waves and establish design wave specification.
2. Assess currents and tidal processes.
3. Formulate sediment budget and perform shoreline evolution analysis.

TOPIC 2: WAVES

- Part 1: Introduction to Ocean Waves
- Part 2: Linear Wave Theory
- Part 3: Nearshore Wave Transformation
- Part 4: Wave Statistics

LEARNING OUTCOMES

Upon completion of this topic, participants should be able:

- To explain generation and dispersion of ocean waves
- To determine the properties of offshore and near-shore waves using linear wave theory.
- To estimate the nearshore wave heights.
- To determine the wave specifications.

PART 1:
INTRODUCTION TO OCEAN WAVES

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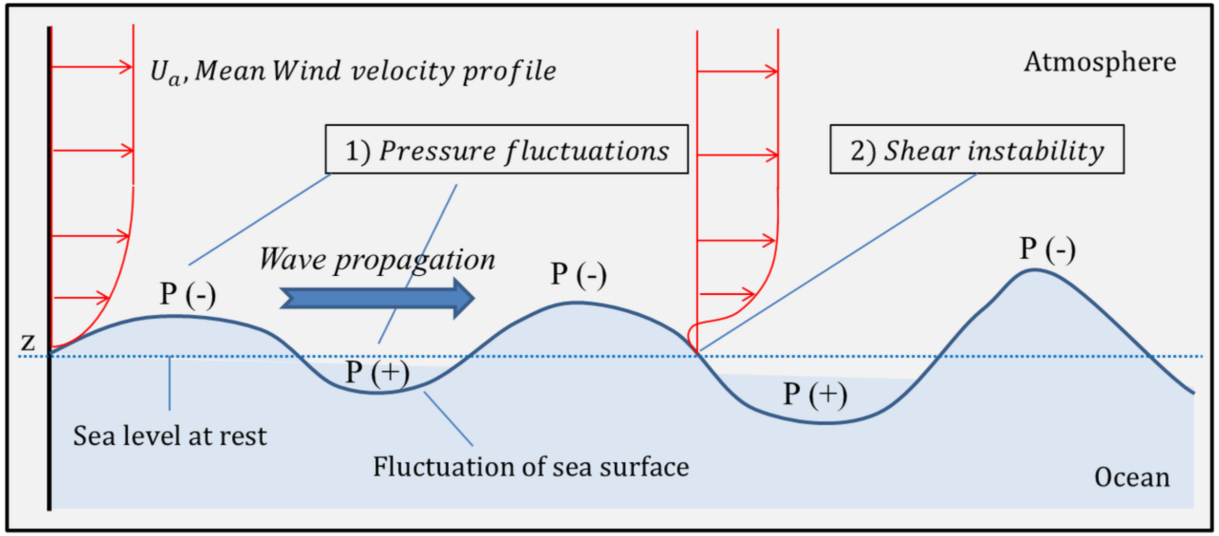
OCEAN WAVES

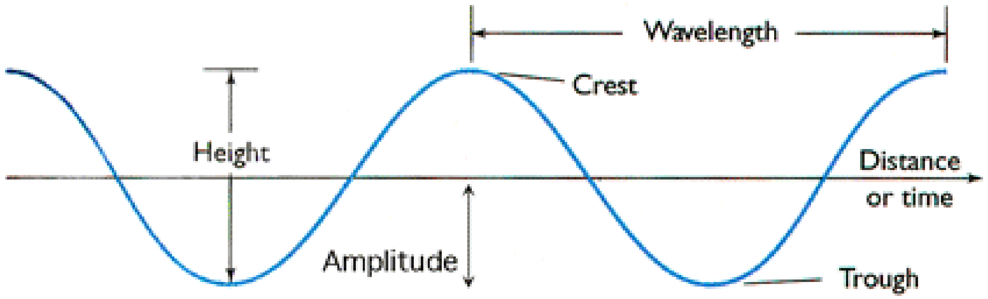


- Waves are disturbances caused by energy moving through water mass.

- An ocean wave is an undulation of the sea surface (usually created by the wind) accompanied by local current, acceleration and pressure fluctuation.
- An ocean wave represents the sea surface in regular motion, as water rises to a wave crest (the highest part of the wave) and sinks to wave trough (the lowest part of the wave).

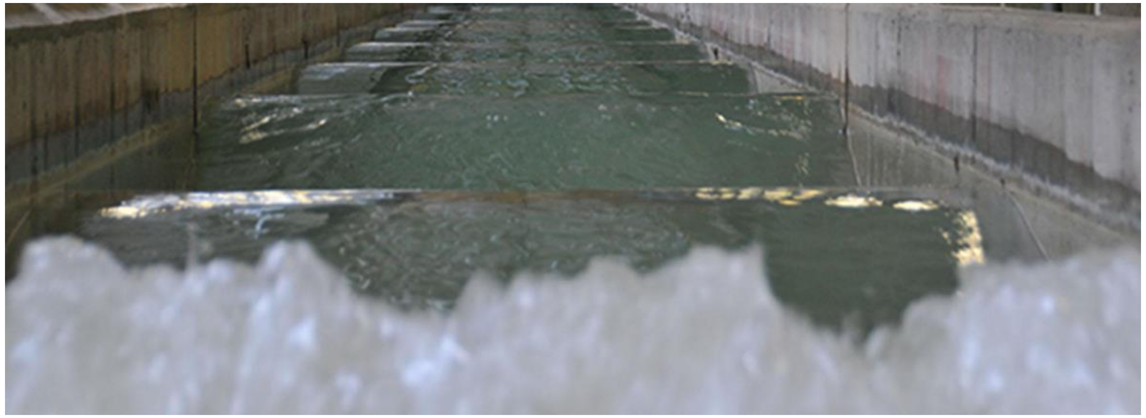
(Source: https://en.wikipedia.org/wiki/Wind_wave#/media/File:Sjyang_waveGeneration.png)

OCEAN WAVES



- The simplest form of waves is sinusoidal, but the actual shape is very complex.
- Knowledge of waves and the forces they generated is essential for the design of coastal & offshore projects.

(Source: https://en.wikipedia.org/wiki/Wind_wave#/media/File:Waves_in_pacifica_1.jpg)



WAVE FORMATION

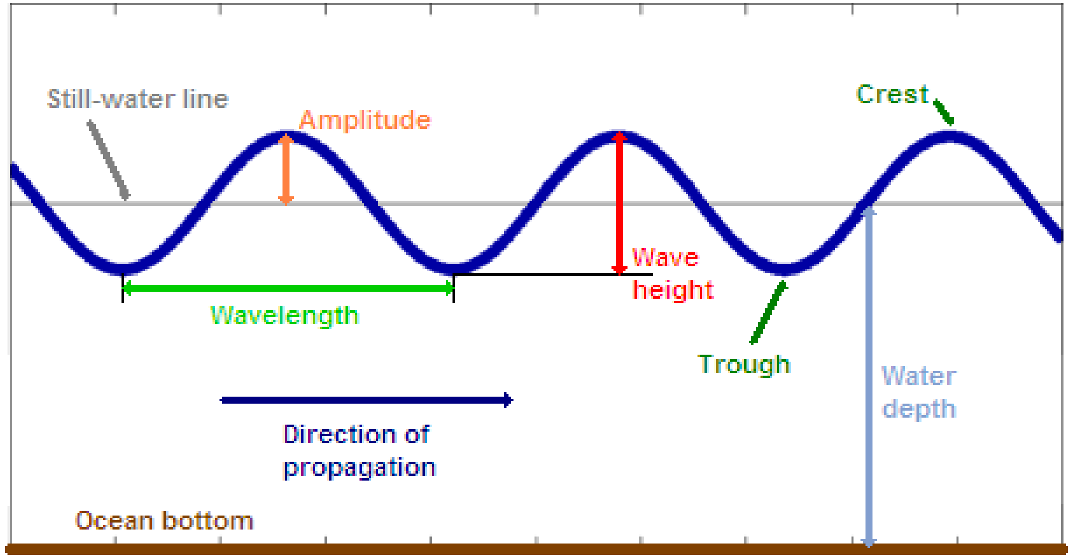
Disturbing forces - Dominant force trying to agitate the still water surface

- Wind
- Displacement - earth quake, landslide, tsunami
- Changes in atmospheric pressure
- Gravitational pull of sun/moon

Restoring forces - Dominant force trying to return the water surface to flat

- Surface tension
- Gravity

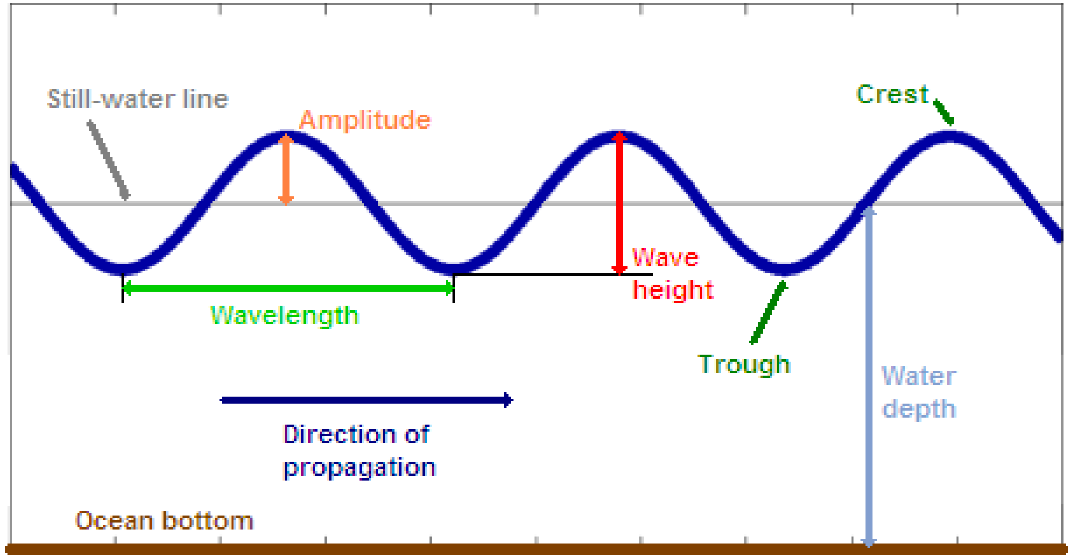
WAVE ANATOMY



(Source: https://cdip.ucsd.edu/m/documents/wave_measurement.html)

- **Still-Water Line (SWL)**: The level of the sea surface if it were perfectly calm and flat.
- **Crest**: The highest point on the wave above the still-water line.
- **Trough**: The lowest point on the wave below the still-water line
- **Depth (d)**: The distance from the ocean bottom to the still-water line.

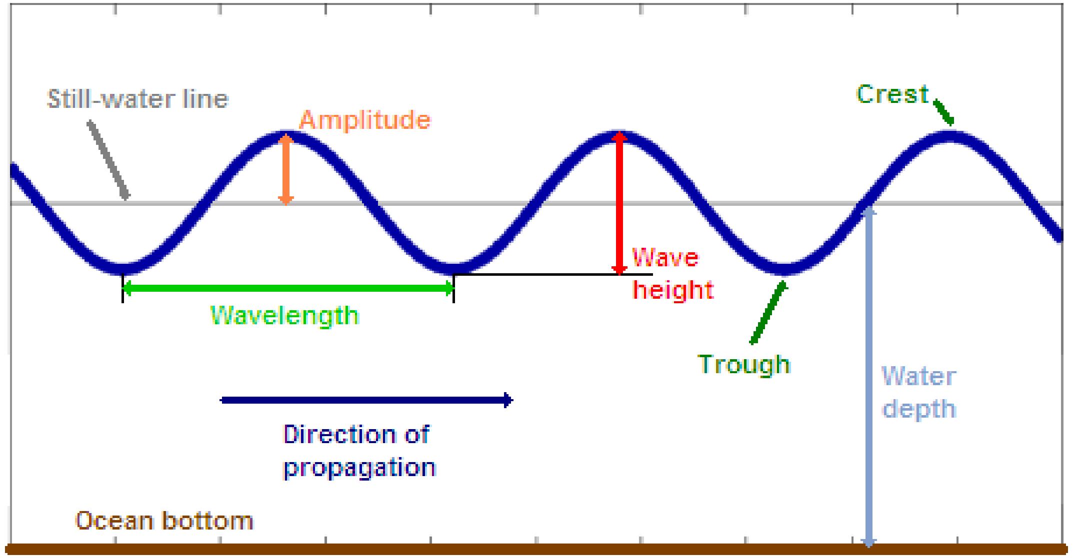
WAVE ANATOMY



(Source: https://cdip.ucsd.edu/m/documents/wave_measurement.html)

- **Amplitude (a):** One-half the wave height or the distance from either the crest or the trough to the still-water line.
- **Wave height (H):** The vertical distance separating the crest from the trough
- **Wavelength (L):** The horizontal distance between the crest of one wave and the crest of an adjacent wave.

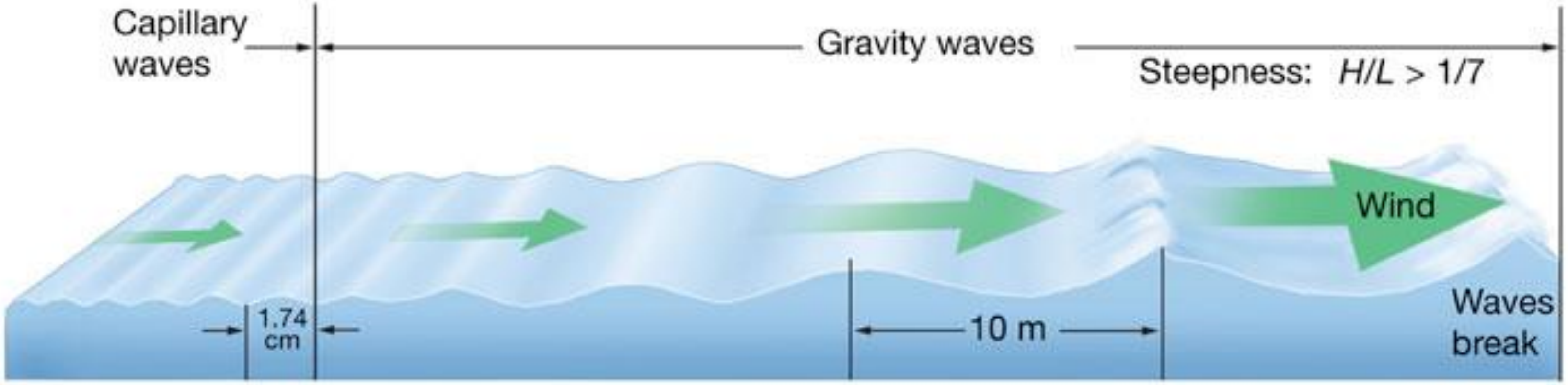
WAVE ANATOMY



(Source: https://cdip.ucsd.edu/m/documents/wave_measurement.html)

- **Wave period (T):** The time it takes two successive crests to pass a fixed point. [Unit: second]
- **Wave frequency (f):** The number of waves passing a point per unit of time. [Unit: Hertz or s^{-1}]

WAVE TYPE



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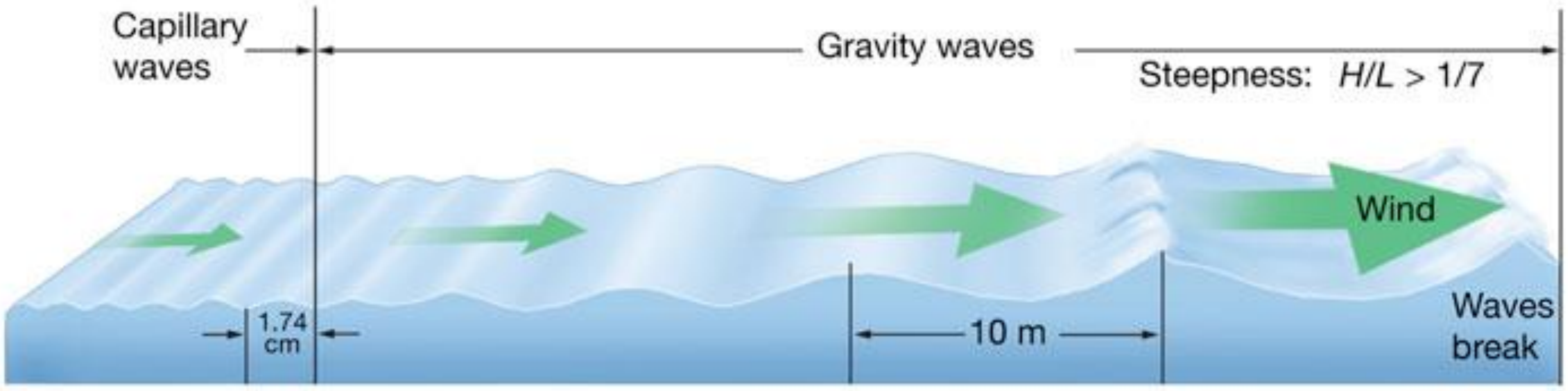
Capillary waves

The smallest water waves which are most readily seen on a flat, calm sea when a puff of wind abruptly disturbs the water surface, creating very tiny, short-lived wavelets.

Swell

A swell is a series of surface gravity waves generated by distant weather systems that propagate thousands of miles across oceans and seas.

WAVE TYPE



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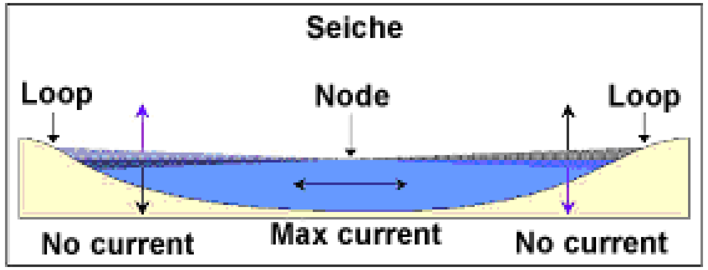
Gravity waves

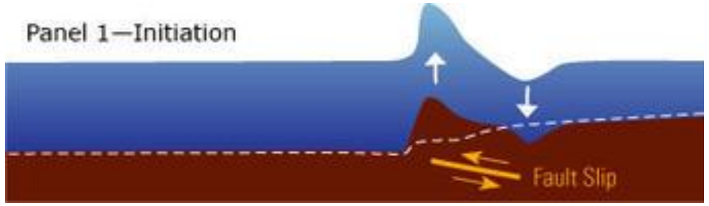
A gravity wave is a wave traveling along the interface between two fluids, whose dynamics are dominated by the effects of gravity. The term 'gravity wave' is typically applied to wind-generated, periodic displacements of the sea surface. Typically, waves on the ocean surface with period of 1-30 s (with concentration of $5 < T < 20$ s), primarily generated by winds.

WAVE TYPE



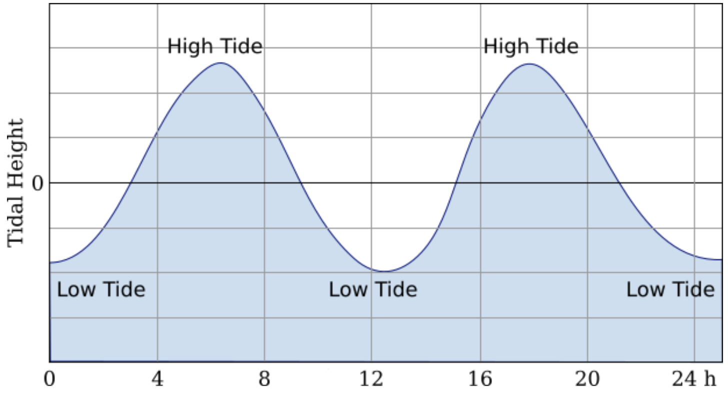
Seiche

Very long period waves with the back-and-forth sloshing of water in harbours.



Tsunami

Very long periods on the order of minutes and tens of minutes are associated with seismic sea waves.



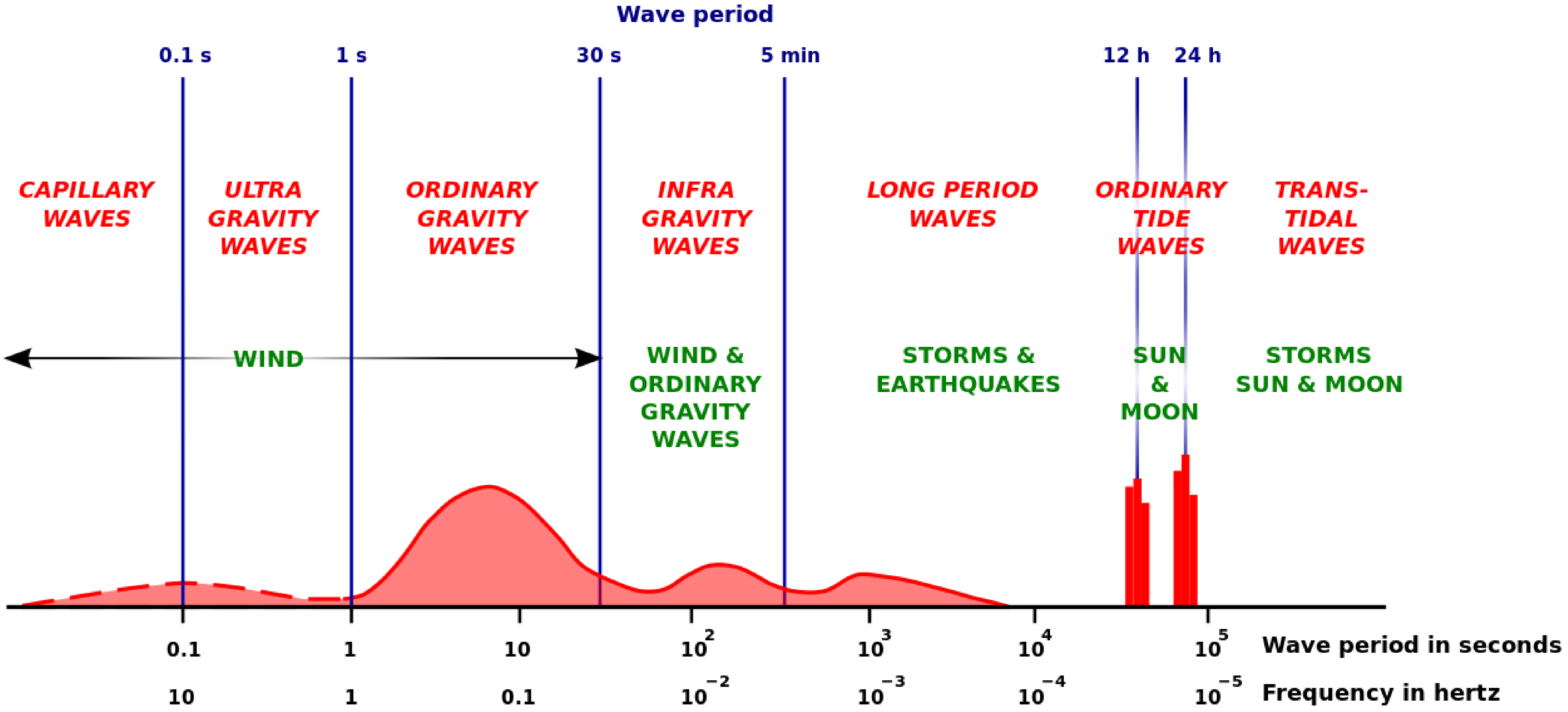
Tide

Shallow water wave that is caused by gravitational attraction of sun and moon.

WAVE TYPE

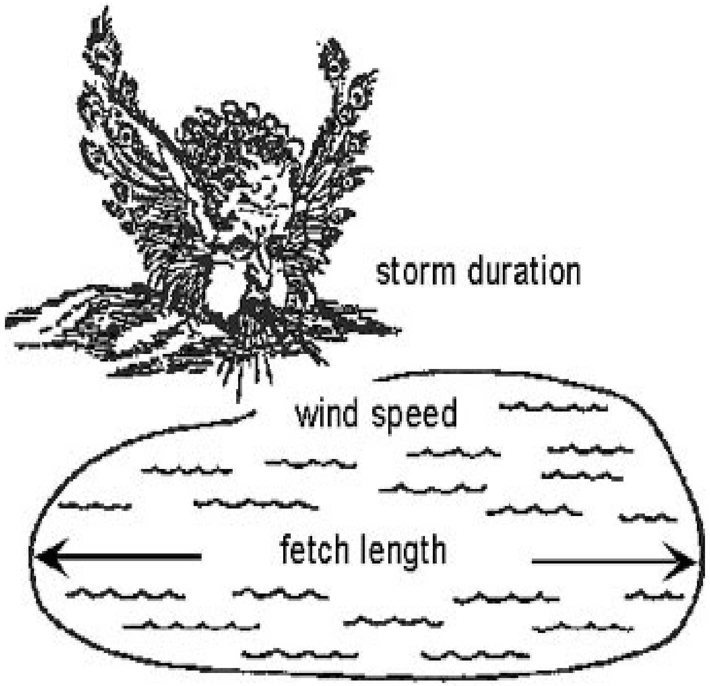
Wave	Period	Wavelength
Capillary waves	< 0.1 s	< 2 cm
Chop	1 – 10 s	1 -10 m
Swell	10 – 30 s	Up to hundreds of m
Seiche	10 min – 10 hr	Up to hundreds of km
Tsunami	10 – 60 min	Up to hundreds of km
Tide	12 – 24 hr	Thousands of km

SPECTRUM OF OCEAN WAVES



(Source: https://en.wikipedia.org/wiki/Wind_wave#/media/File:Munk_ICCE_1950_Fig1.svg)

WAVE GENERATION AREA



When wind blows, ripples are generated within the storm area.

The stronger the wind, the larger the waves. As wind speed increases, so do the wavelength, the period, and the height of the resulting waves.

The variety and size of wind-generated waves are controlled by:

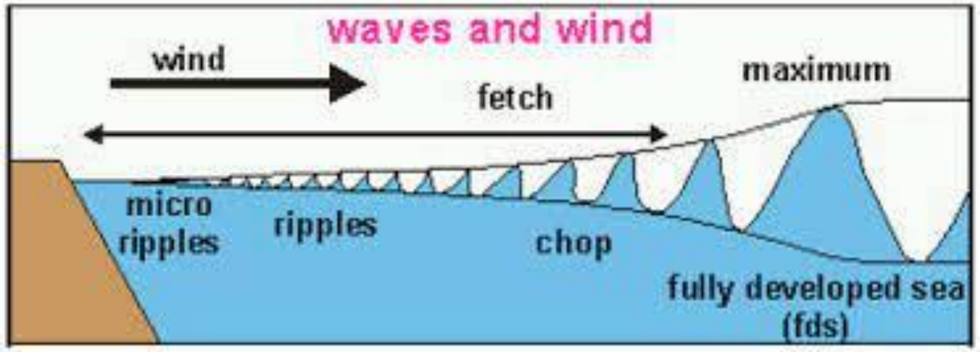
1. Wind speed
2. Wind duration
3. Fetch

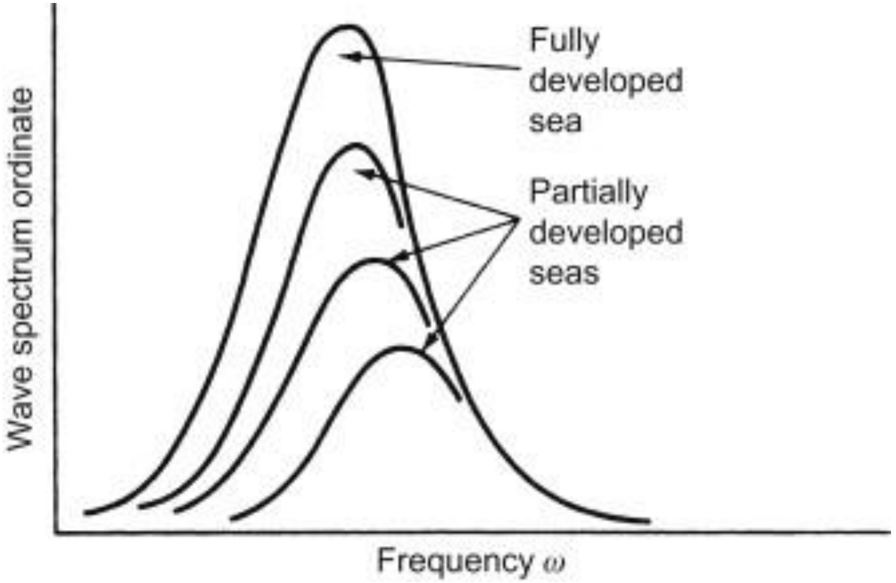
Fetch – Length of the area of water over which the wind blows

In the wave generation area, energy is transferred from shorter period waves to the longer waves.

FULLY DEVELOPED SEA

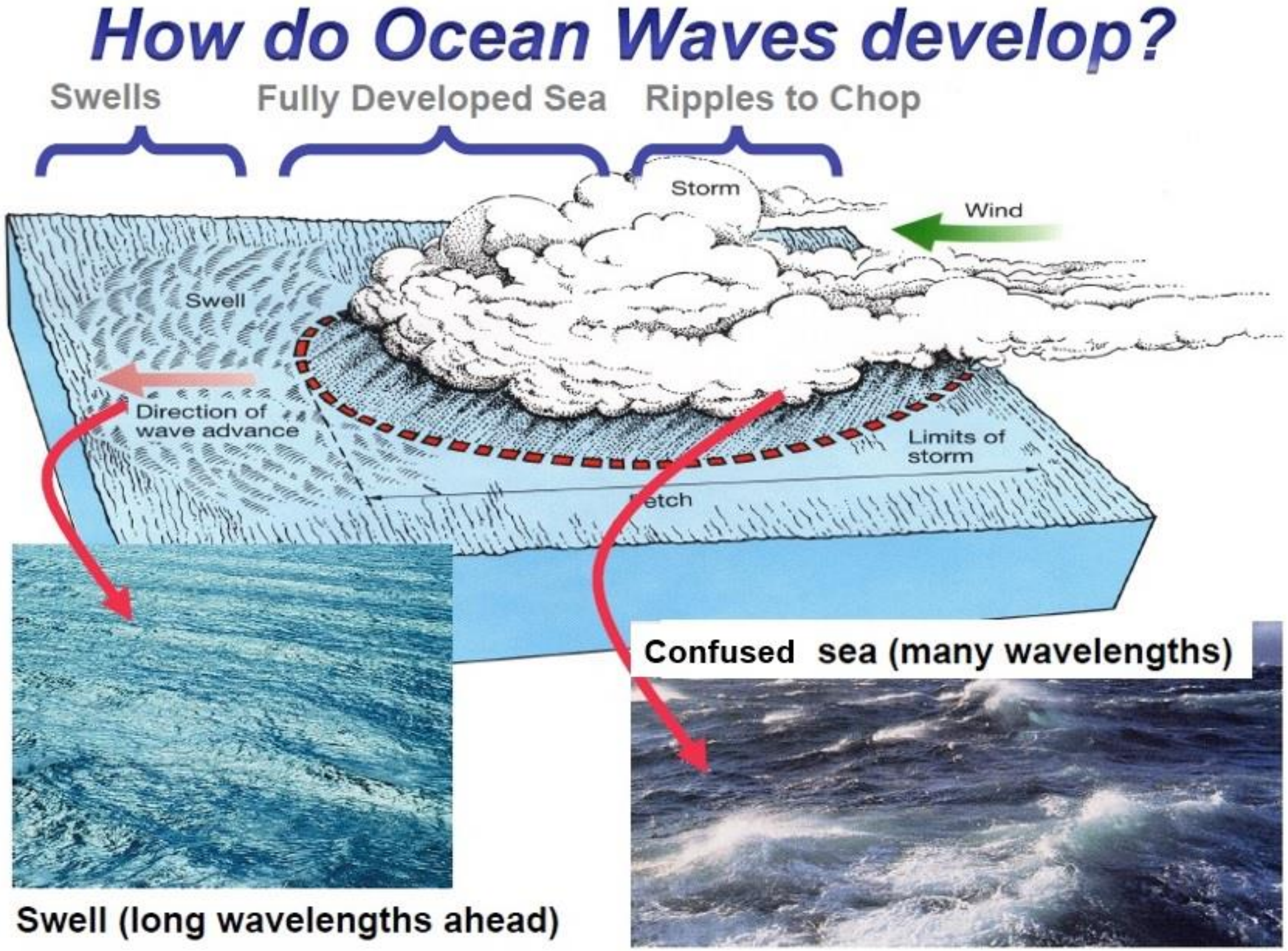
- Waves will continue to grow in size until they reach a maximum size that is determined by the wind speed and fetch.
- At these stage of wave growth, waves stop growing in size under the existing wave conditions because the energy supplied by the wind equals the energy lost by wave breaking and leaving the fetch area.
- This sea at this state is termed a fully developed sea condition.
- Fully developed sea is the maximum height of waves produced by conditions of wind speed, duration and fetch.





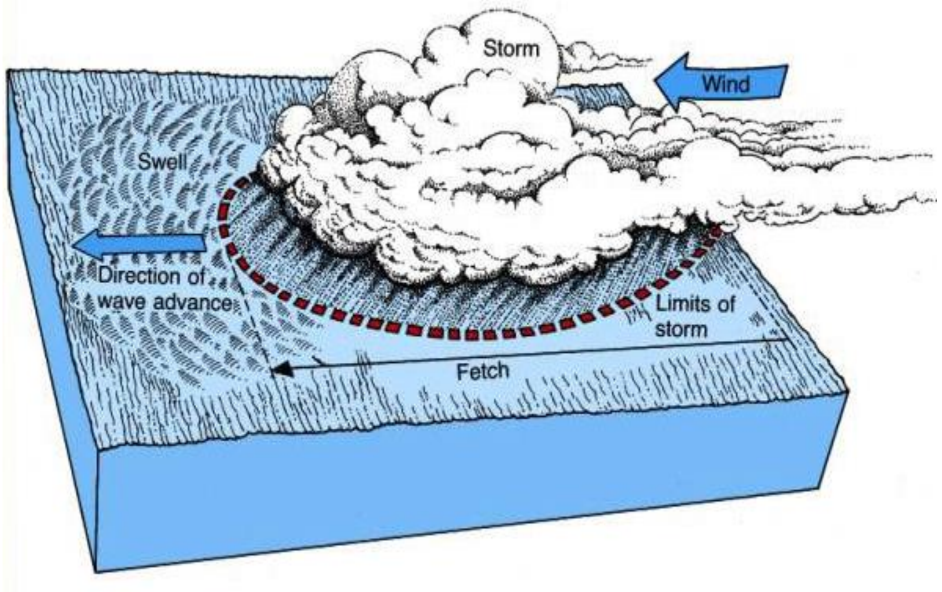
(Source: https://islandwatersports.com/blog/surf-science-why-do-waves-break/; https://www.sciencedirect.com/topics/engineering/ship-model-tanks)

WAVE PROPAGATION



(Source: https://www.eoas.ubc.ca/courses/atsc113/sailing/met_concepts/08-met-waves/8b-wave-characteristics/index.html)

SWELL WAVES



Those waves traveled out of the generation area is called SWELL. They appear to be almost unidirectional and long crested (i.e., they have well-defined and distinctly separated crests). The speed of the swells is faster than wind speed outside the storm area. The wave steepness decreases as they run over long distance with minimum energy loss.

Swells will eventually form groups or trains of waves, which travel at ½ speed of individual waves . The swells can travel hundreds or thousands of kilometers without much loss of energy. The energy is dissipated internally within the fluid, by interaction with the air above, by turbulence upon breaking and by percolation and friction with the seabed. Short-period waves lose their energy more rapidly than the long-period waves.

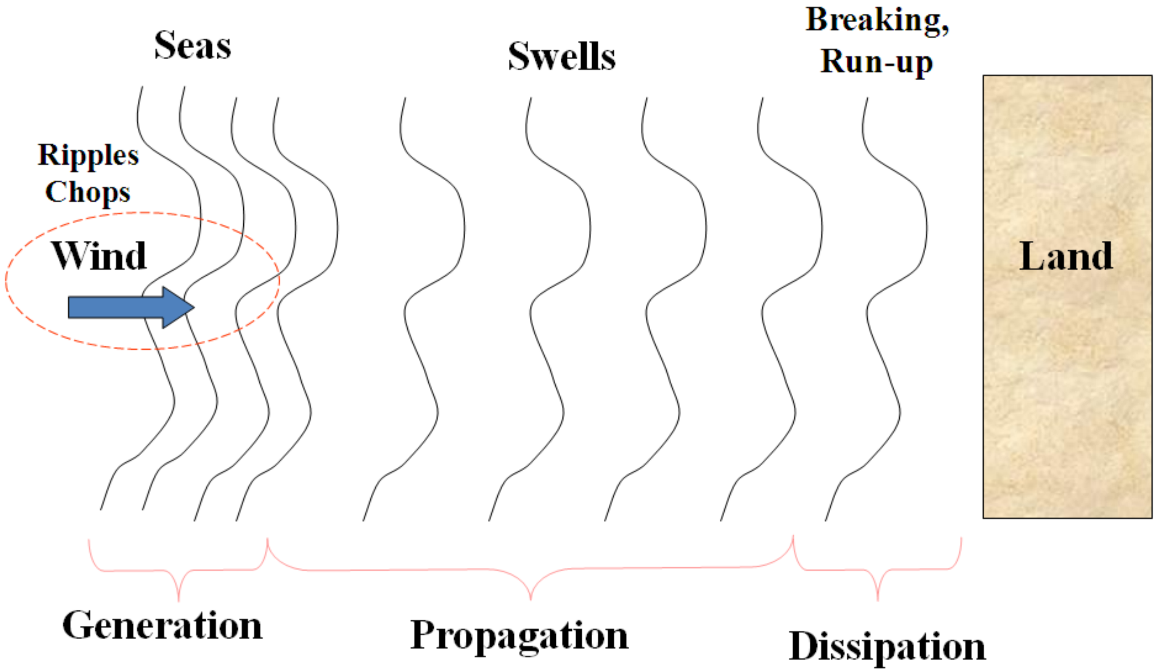
SURFACE WAVES

Seas	Swells
 	 

SEAS VS. SWELLS

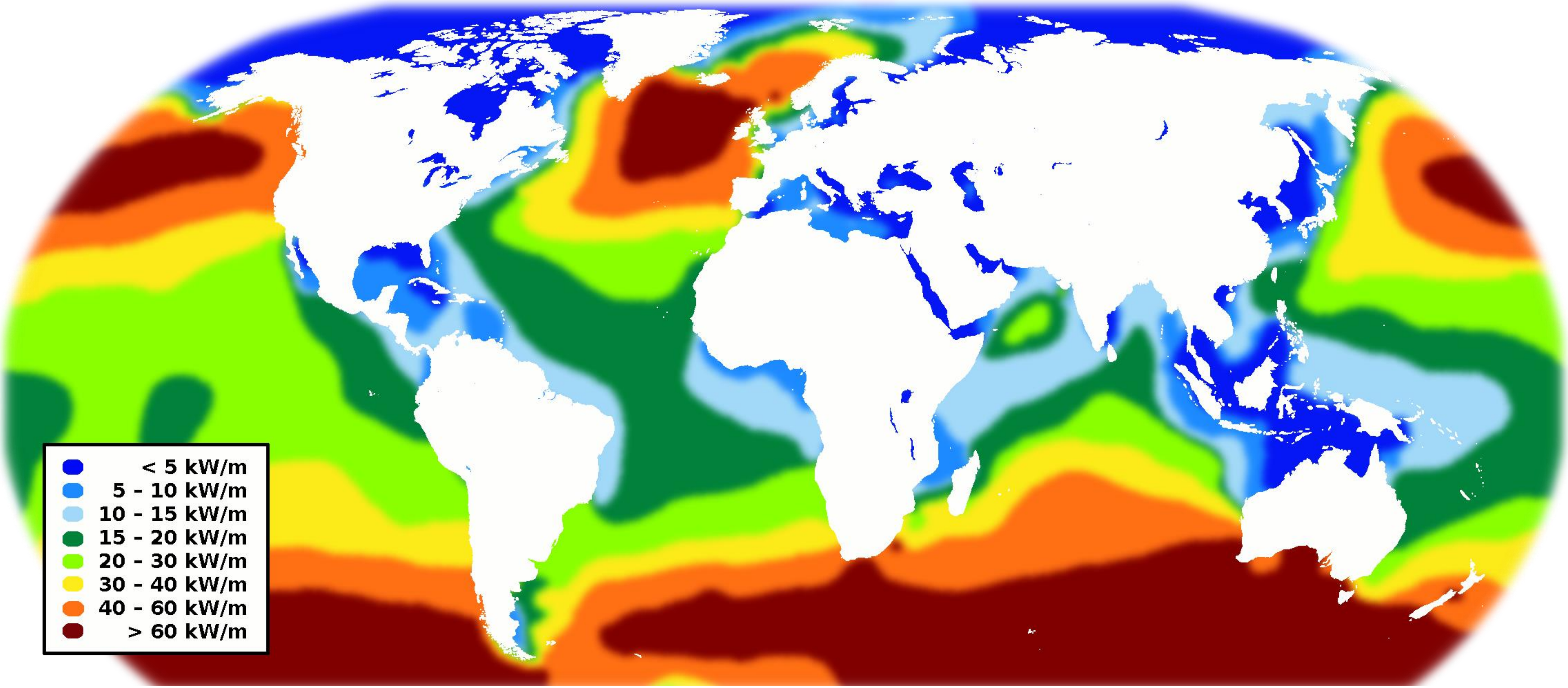
Seas	Swells
Short-period waves created by winds	Waves that have moved out of the generation /storm area
More disturbed sea surface with choppy waves of mixed wavelengths (periods) and different wave heights; Irregular waves with short-crested and their periods are within 3 – 25 sec	Waves are in more orderly state with definite crests and troughs; Regular waves with well-defined long crests and relatively long periods, i.e. greater than 10 sec
Waves of high steepness; short wavelength L = 10-20 H	Waves of mild steepness; Long wavelength L = 30-500 H
Present if windy	Present even with no wind
Propagate in the wind direction	Propagate in groups

WAVE DISPERSION



Although waves of different periods existed originally together in the generation area (seas), in time the various wave components in the sea separate from one another. Longer period waves move faster and reach distant sites first, and shorter period waves may reach the site several days later. This process is called WAVE DISPERSION.

WORLD WAVE ENERGY RESOURCES



Source: https://en.wikipedia.org/wiki/File:World_wave_energy_resource_map.png

WAVE ENERGY CONVERTERS

https://www.youtube.com/watch?v=sZuc4LMtHoY

PART 2:
LINEAR WAVE THEORY

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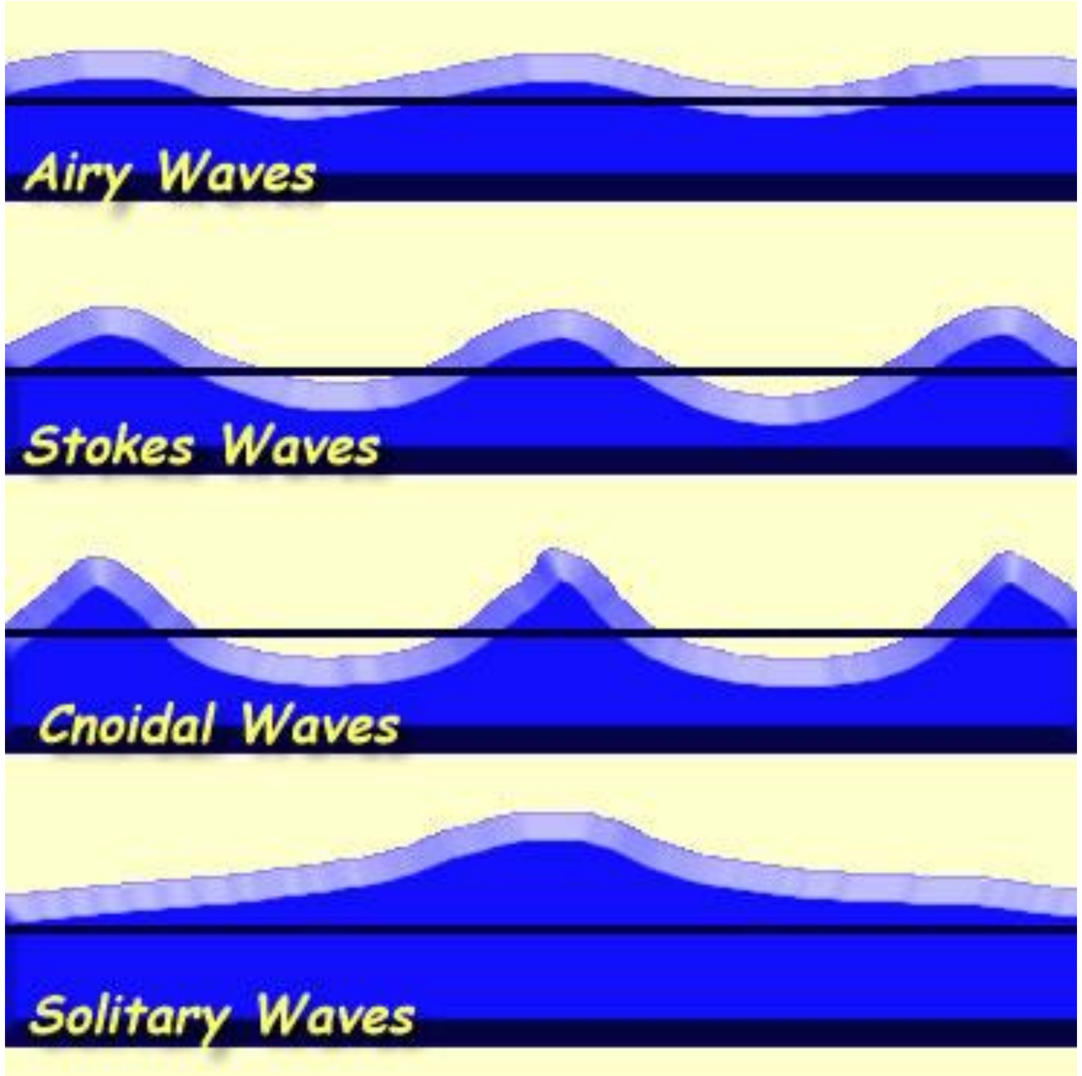
WAVE THEORIES



- Numerous water wave theories applicable to different environments dependent upon the specific environmental parameters, e.g., water depth, wave height and wave period.
- All ocean wave theories assume that the waves are periodic uniform, having a period *T* and height *H*.

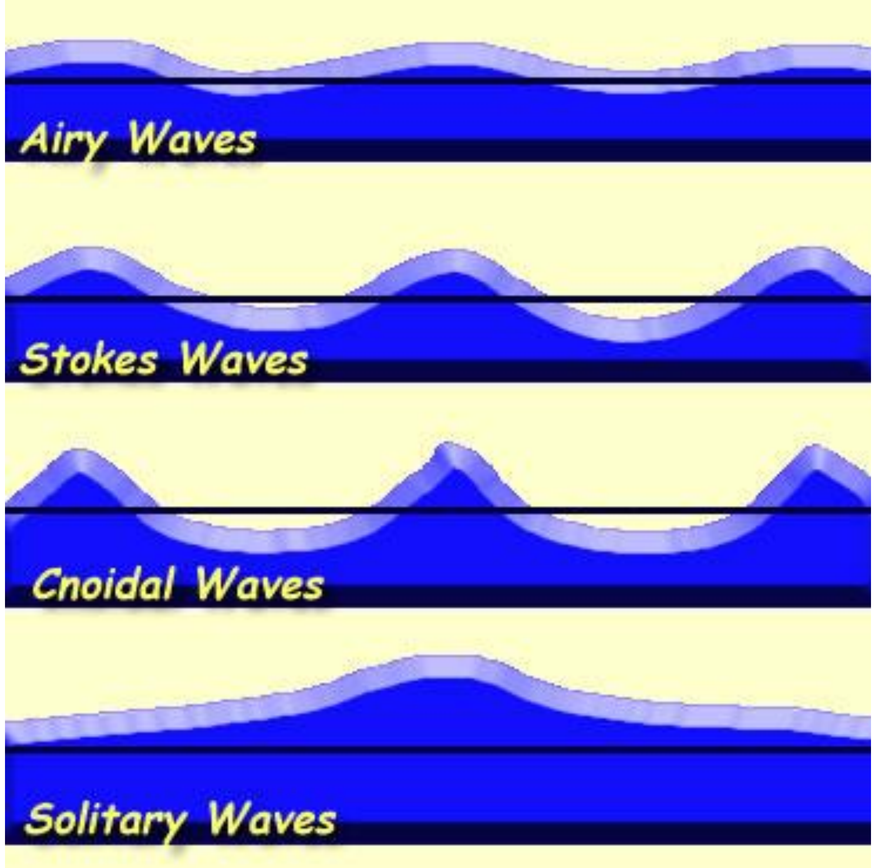
WAVE THEORIES

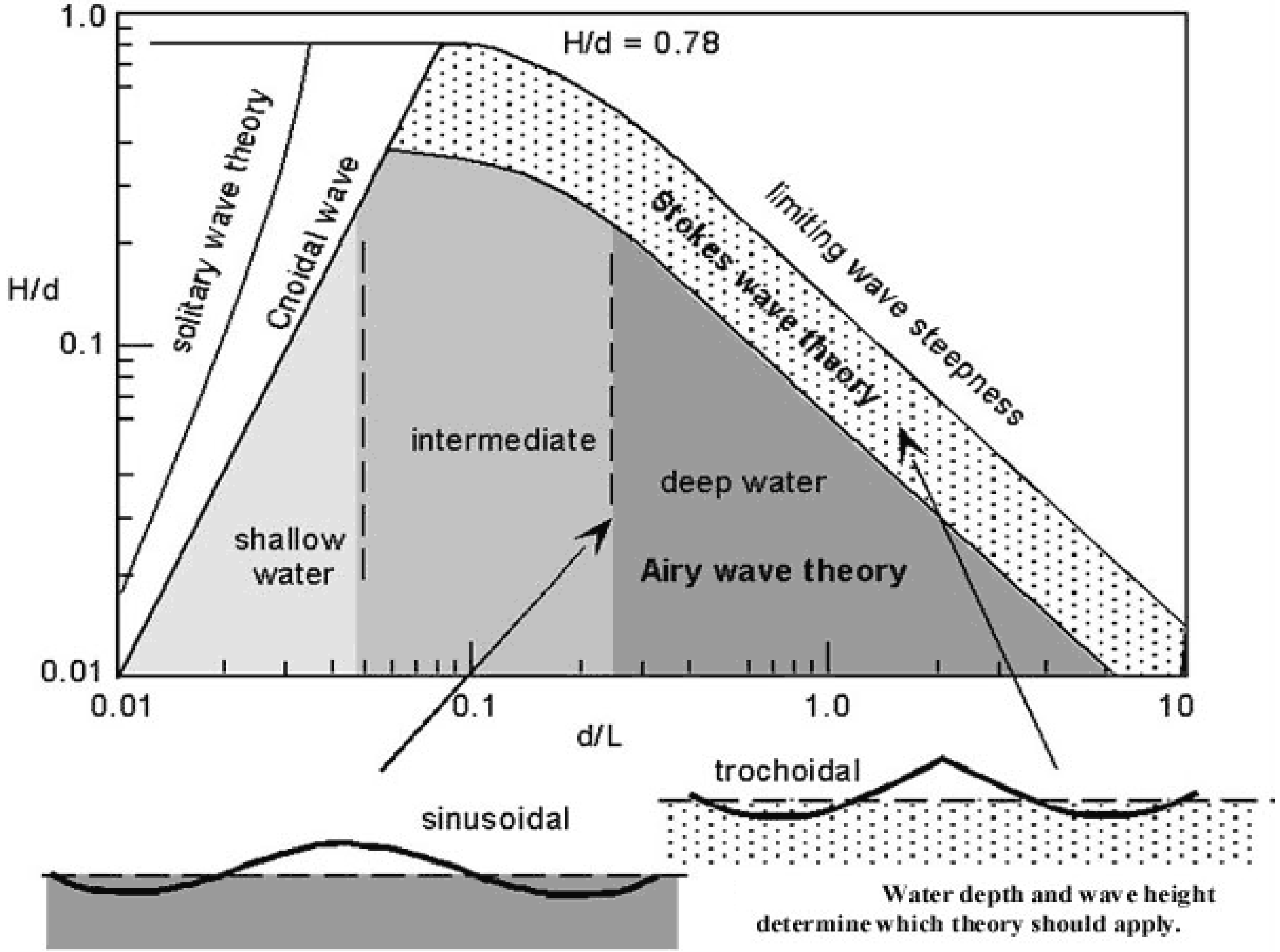
- Waves at sea are very complex due to irregularity of wave shape.
- Several theories exist to describe wave behaviour:

Wave Theory	Reference	Water Condition
Linear wave theory	Airy (1845)	Deep water ($d/L_o > 0.5$)
Stoke wave theory	Stokes (1847) Fenton (1985)	
Cnoidal wave theory	Korteweg & De Vries (1895) Keulegan & Patterson (1940) Svendsen (1974) Fenton (1979)	Transitional water ($0.16 < d/L_o > 0.5$)
Solitary wave theory	Boussinesq, 1872 McCowan (1981) Grimshaw (1971) Fenton (1972)	Shallow water ($d/L_o < 0.1$)

Note: Relative water depth, d/L_o, where d = water depth; L_o = deepwater wavelength

WAVE THEORIES

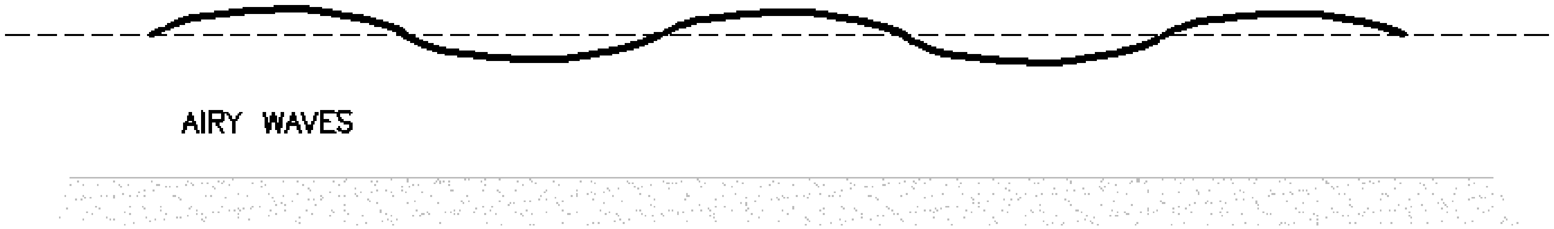




Water depth and wave height determine which theory should apply.

Source: https://slidetodoc.com/department-of-marine-sciences-school-of-environment-coastal-4/

LINEAR WAVE THEORY



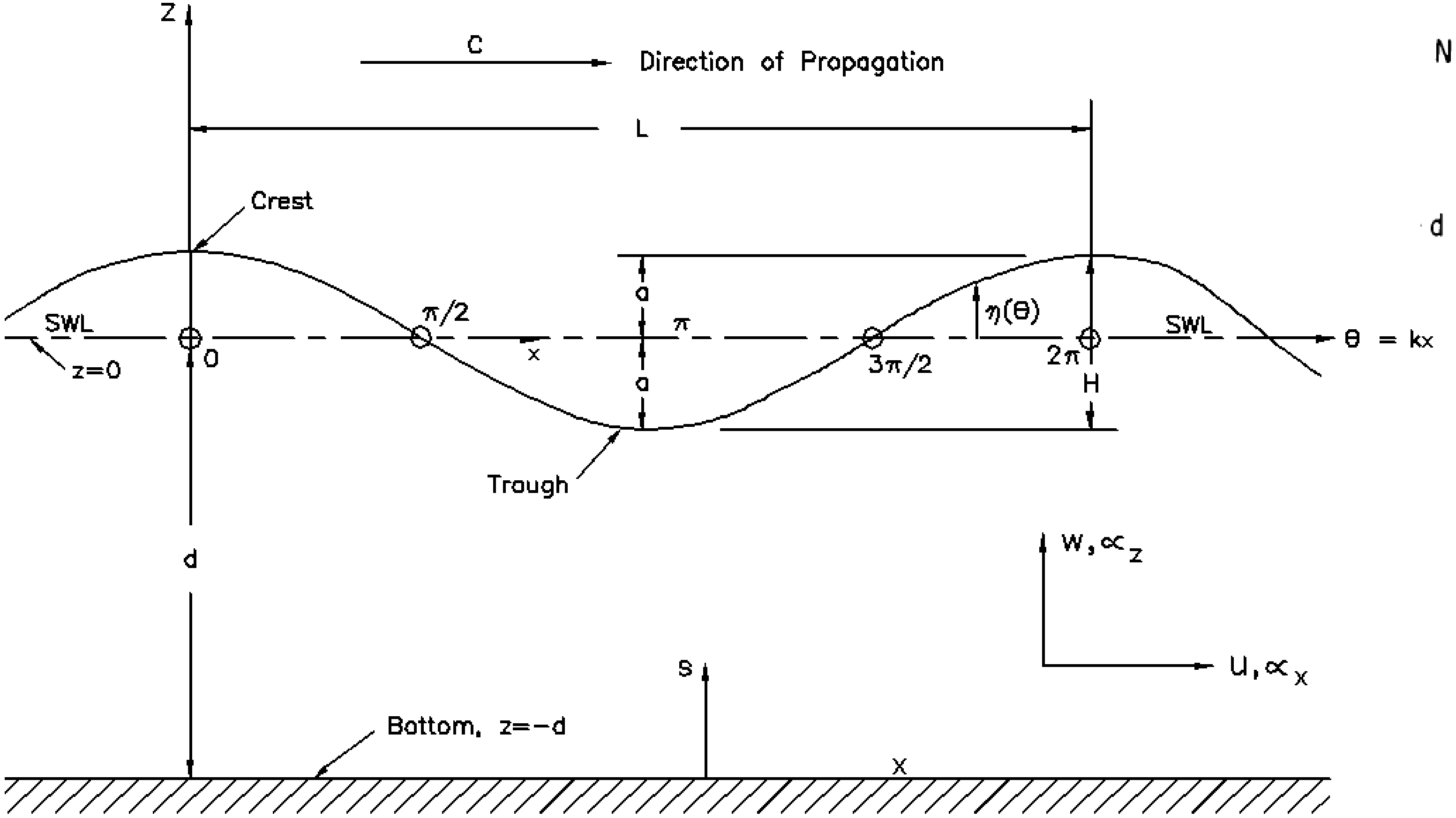
- The simplest wave theory is the *first-order*, *small-amplitude*, or *Airy* wave theory which will hereafter be called ***linear wave theory***.
- The basis for the wave theory is the sinusoidal wave, and it constitutes the 1st order of approximation of the Stokes' theory.
- Most commonly used wave theory due to less mathematically complex.
- Both crest and trough amplitudes must be equal.
- Most accurate for low amplitude waves in deep water ($H<<L$); less accurate for predicting wave behavior in shallow water.
- Can be applied to both sea and swell but best to characterize swell wave with its small steepness (H/L).

ASSUMPTIONS

1. The fluid is homogeneous and incompressible (*i*.e. constant density)
2. Surface tension can be neglected
3. Pressure at the free surface is uniform and constant
4. The fluid is inviscid
5. The flow is irrotational so that water particles do not rotate
6. The particular wave being considered does not interact with any other water motions.
7. The seabed is a horizontal, fixed & impermeable boundary (i.e. the vertical velocity at the bed is zero)
8. Wave amplitude is small compared to the length and water depth
9. The waveform is invariant in time and space
10. Waves are plane or long crested (2-dimensional)

SYMBOLS

The wave profile is simplified to a linear, sinusoidal wave form:



NOTE : (a) $\eta = a \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$

(b) For given origin (x = o) wave profile is shown for t = 3T/4, 7T/4, 11T/4 ……

(c) η = a = H/2 at wave crest

η = − a = − H/2 at wave trough

H = Wave height
a = Wave amplitude = H/2
T = Wave period
L = Wavelength
z = Elevation
C = Wave celerity
d = Water depth
η= Displacement of water surface relative to SWL

DIMENSIONLESS PARAMETERS

Angular frequency, $\omega = 2\pi/T$

Wave number, $k = 2\pi/L$

Wave celerity or phase velocity, $C = L/T = \omega/k$

Wave steepness, $\varepsilon = H/L$

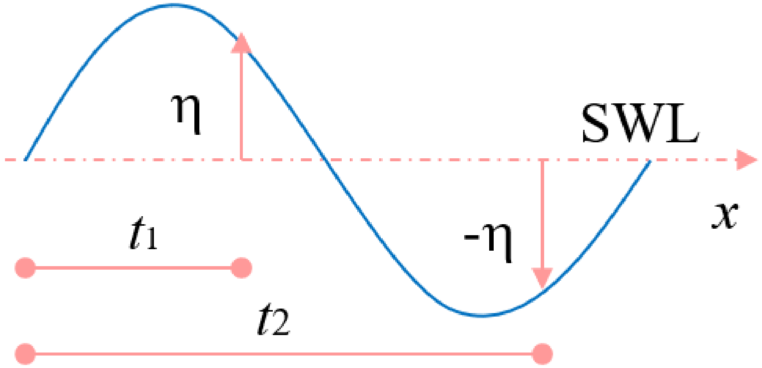
Relative depth, d/L

Relative wave height, H/d

WAVE PROFILE

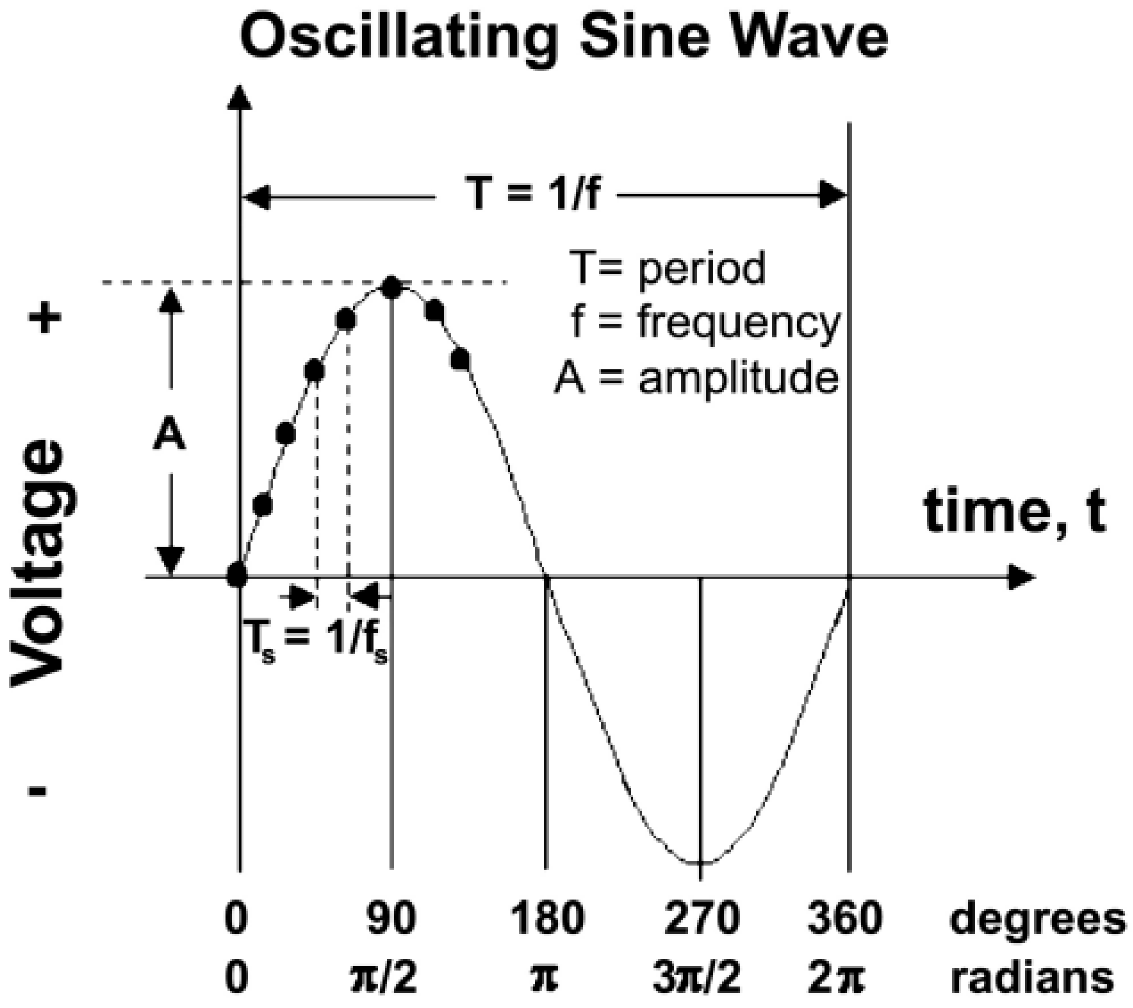
The displacement of the sinusoidal water surface relative to the SWL (η) is described by

$$\eta = a \cos (kx - \omega t) = \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) = a \cos \theta$$



a = the amplitude of the wave = $H/2$
x = distance in the direction of wave propagation
t = time (different from wave period, T)
k = wave number = $2\pi/L$
ω = angular wave frequency = $2\pi/T$

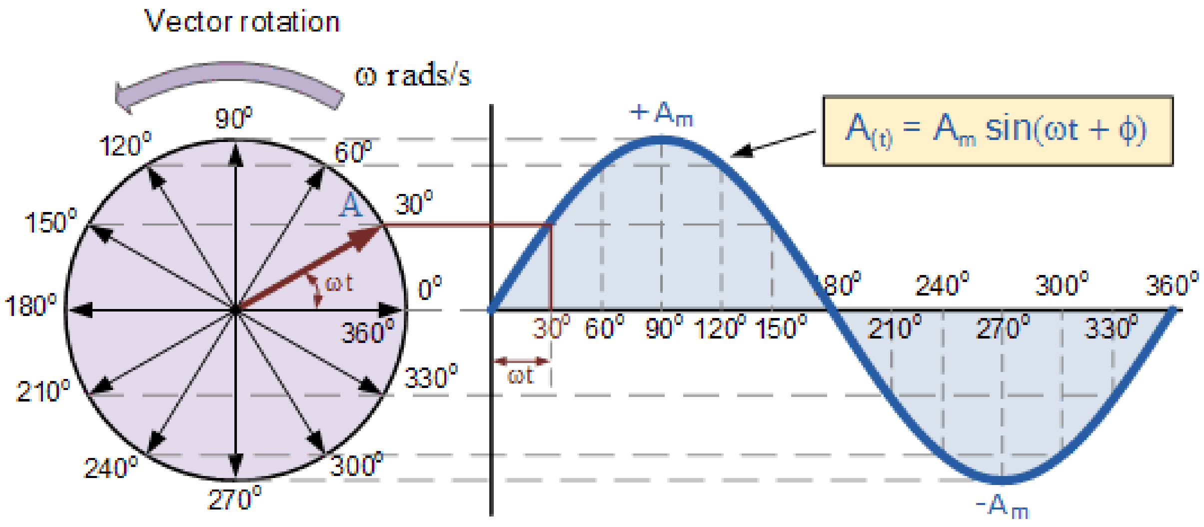
PHASE ANGLE



Plot of one cycle of a sinusoidal function. The phase for each argument value, relative to the start of the cycle, in degrees from 0° to 360° and in radians from 0 to 2π.

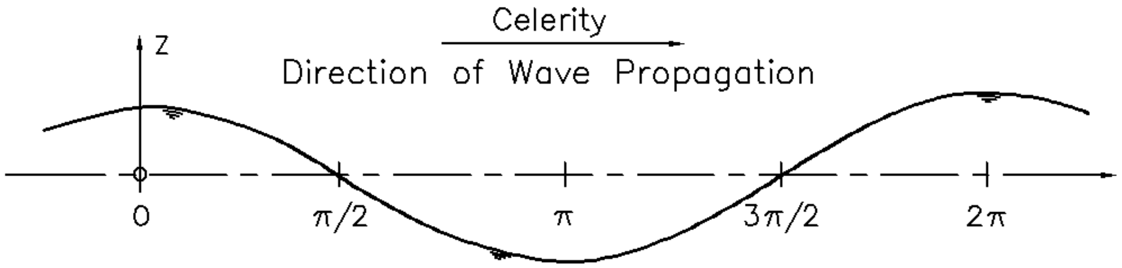
PHASE ANGLE

A point in the period to which the wave motion has advanced with respect to a given initial reference point.



Rotating Phasor

Sinusoidal Waveform in the Time Domain



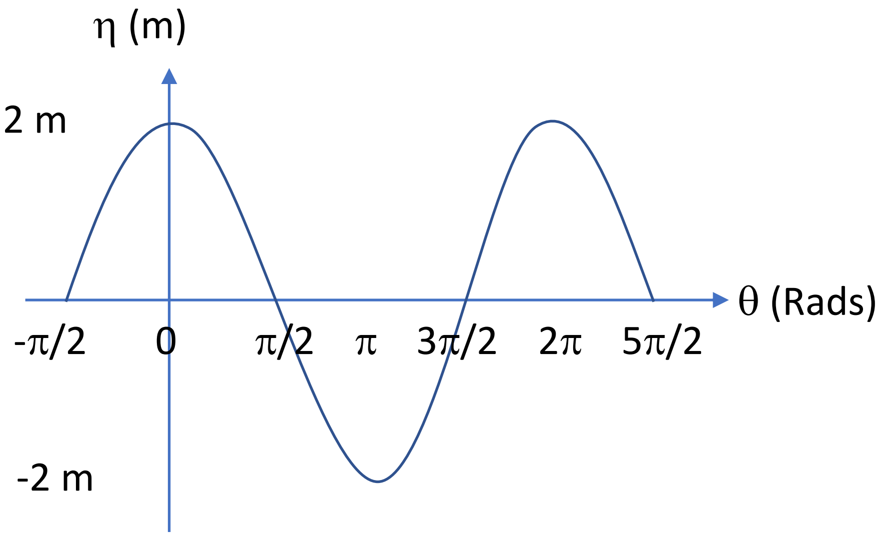
$$\eta = a \cos(kx - \omega t) = \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) = a \cos \theta$$

PROBLEM 1

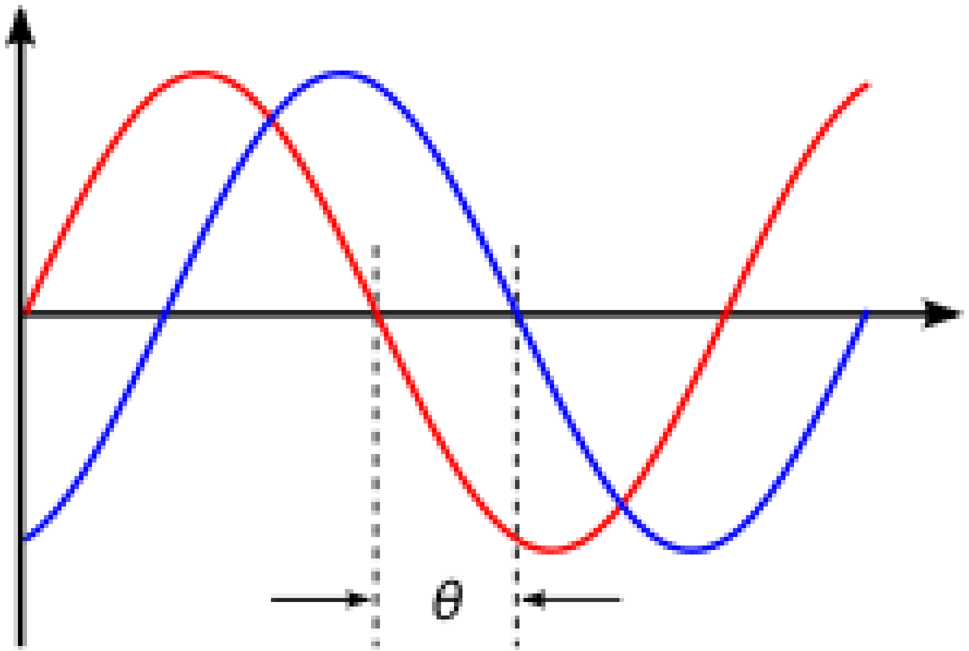
Construct a wave profile for a wave of 4 m high

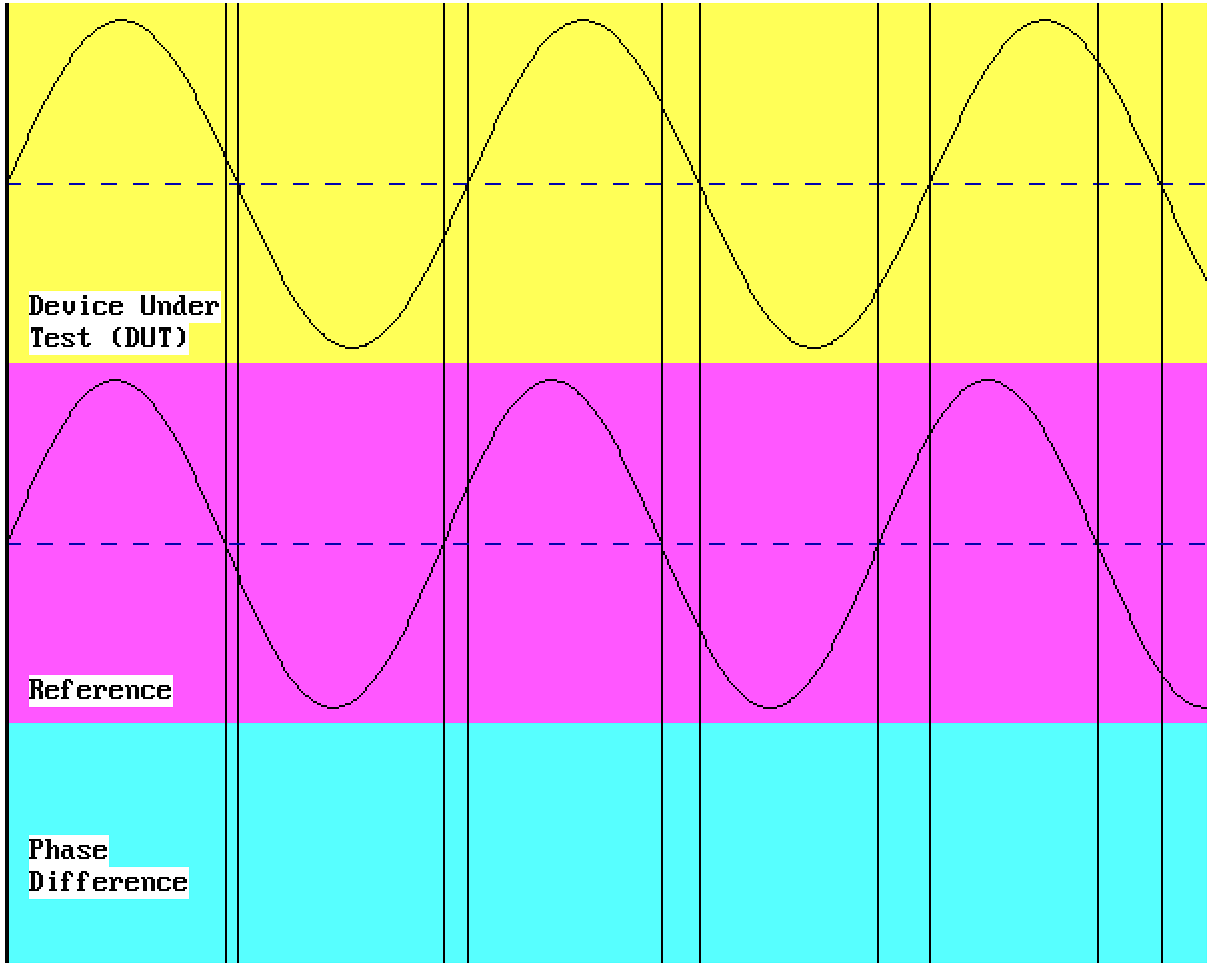
$$\eta = a \cos(kx - \omega t) = \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) = a \cos \theta$$

θ	Cos θ	η = a cos θ
-π/2	0	0
0	1	2
π/2	0	0
π	-1	-2
3π/2	0	0
2π	1	2
5π/2	0	0



PHASE DIFFERENCE/ PHASE SHIFT





WAVE CELERITY

Since the distance traveled by a wave during one wave period, *T* is equal to one wavelength, *L*, wave celerity, *C* can be expressed as:

$$C = \frac{L}{T} \qquad \textbf{(1)}$$

An expression relating wave celerity to wavelength, *L* and water depth, *d* (known as the linear dispersion relation) is given by:

$$C = \sqrt{\frac{gL}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)} \qquad \textbf{(2)}$$

or

$$C = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right) \qquad \textbf{(3)}$$

WAVELENGTH

From Eq. (1) and Eq. (3), an expression for wavelength as a function of depth and wave period may be obtained as

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right) = \frac{gT}{\omega} \tanh(kd) \qquad \textbf{(4)}$$

Use of Eq. (4) involves some difficulty since the unknown L appears on both sides of the equation.

Tabulated values of d/L and d/L_o in Table C-1 of Shore Protection Manual (1984) may be referred to determine the wavelength, L in Eq. (4).

[Note: L_o is the deepwater wavelength]

Table C-1 (Shore Protection Manual, 1984)

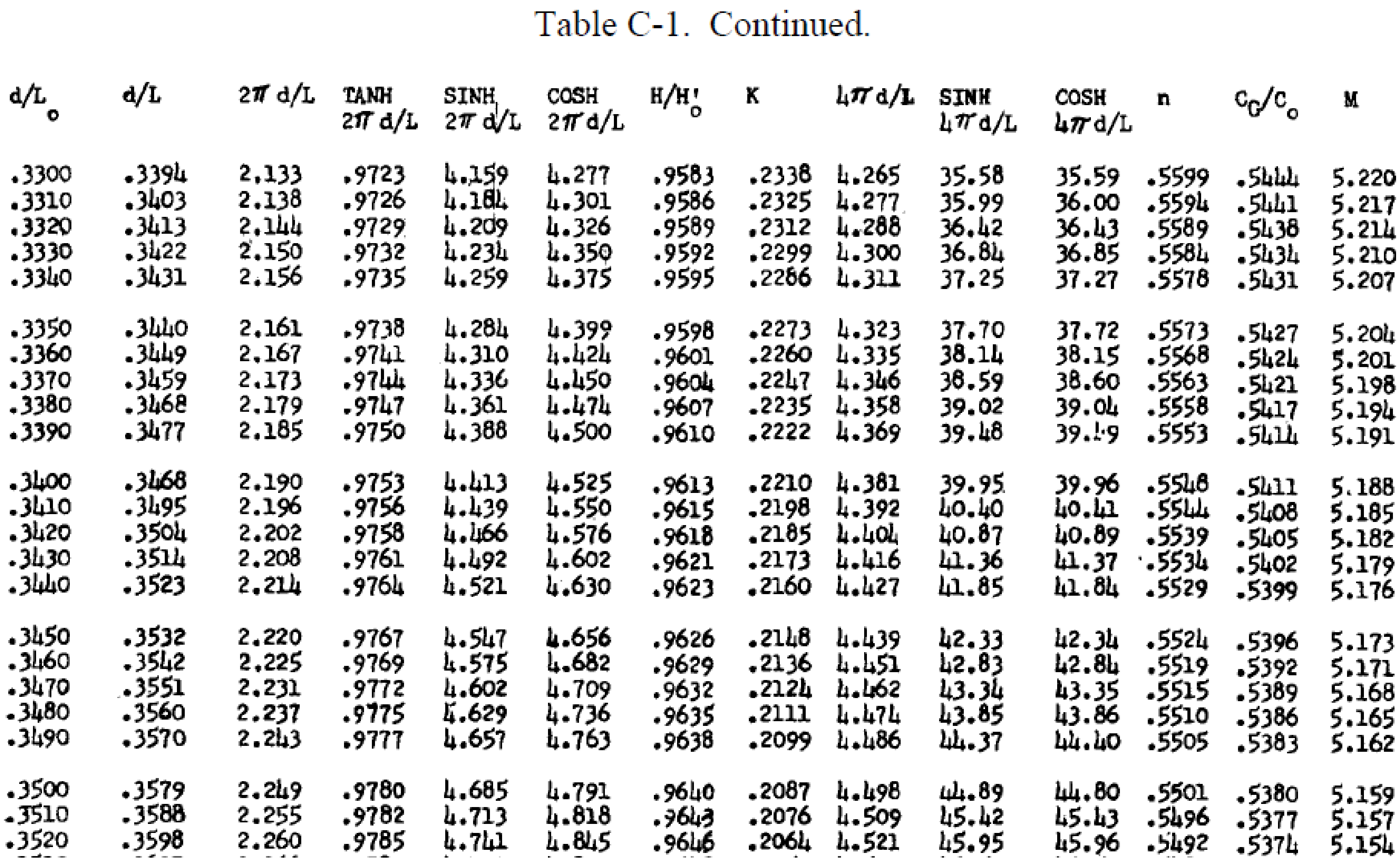


Table C-1. Continued.

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H_o'	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.3300	.3394	2.133	.9723	4.159	4.277	.9583	.2338	4.265	35.58	35.59	.5599	.5444	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.9586	.2325	4.277	35.99	36.00	.5594	.5441	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.9589	.2312	4.288	36.42	36.43	.5589	.5438	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.9592	.2299	4.300	36.84	36.85	.5584	.5434	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.9595	.2286	4.311	37.25	37.27	.5578	.5431	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.9598	.2273	4.323	37.70	37.72	.5573	.5427	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.9601	.2260	4.335	38.14	38.15	.5568	.5424	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.9604	.2247	4.346	38.59	38.60	.5563	.5421	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.9607	.2235	4.358	39.02	39.04	.5558	.5417	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.9610	.2222	4.369	39.48	39.49	.5553	.5414	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.9613	.2210	4.381	39.95	39.96	.5548	.5411	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.9615	.2198	4.392	40.40	40.41	.5544	.5408	5.185
.3420	.3504	2.202	.9758	4.466	4.576	.9618	.2185	4.404	40.87	40.89	.5539	.5405	5.182
.3430	.3514	2.208	.9761	4.492	4.602	.9621	.2173	4.416	41.36	41.37	.5534	.5402	5.179
.3440	.3523	2.214	.9764	4.521	4.630	.9623	.2160	4.427	41.85	41.84	.5529	.5399	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.9626	.2148	4.439	42.33	42.34	.5524	.5396	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.9629	.2136	4.451	42.83	42.84	.5519	.5392	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.9632	.2124	4.462	43.34	43.35	.5515	.5389	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.9635	.2111	4.474	43.85	43.86	.5510	.5386	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.9638	.2099	4.486	44.37	44.40	.5505	.5383	5.162
.3500	.3579	2.249	.9780	4.685	4.791	.9640	.2087	4.498	44.89	44.80	.5501	.5380	5.159
.3510	.3588	2.255	.9782	4.713	4.818	.9643	.2076	4.509	45.42	45.43	.5496	.5377	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.9646	.2064	4.521	45.95	45.96	.5492	.5374	5.154

WAVELENGTH

Eckart (1952) gives an approximate expression for Eq. (4), which is correct to within about 10%:

$$L \approx \frac{gT^2}{2\pi}\sqrt{\tanh\left(\frac{4\pi^2}{T^2}\frac{d}{g}\right)} \qquad \textbf{(5)}$$

which can be written as

$$L \approx L_o\sqrt{\tanh\left(\frac{2\pi d}{L_o}\right)} \qquad \textbf{(6)}$$

WAVE CLASSIFICATION BY RELATIVE DEPTH

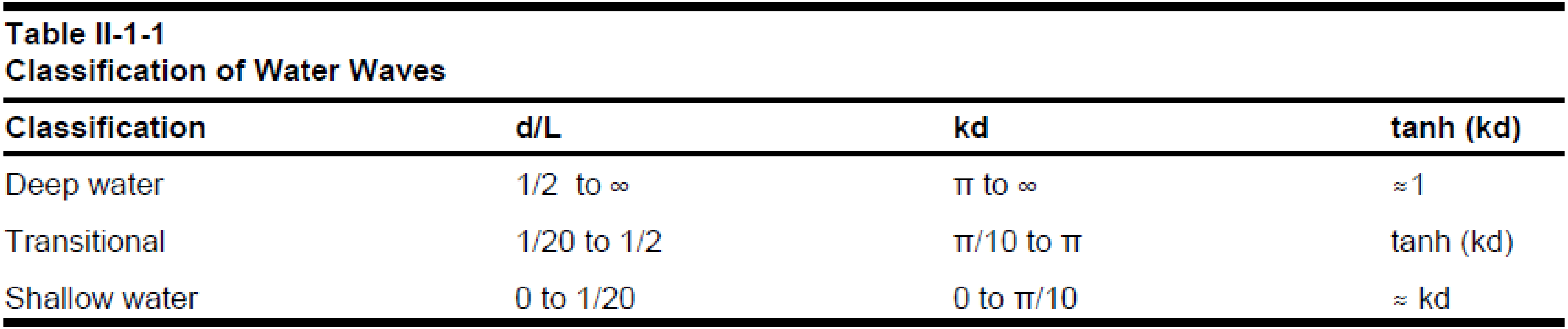


Table II-1-1
Classification of Water Waves

Classification	d/L	kd	tanh (kd)
Deep water	1/2 to ∞	π to ∞	≈ 1
Transitional	1/20 to 1/2	π/10 to π	tanh (kd)
Shallow water	0 to 1/20	0 to π/10	≈ kd

$$\boxed{L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)}$$

For large d, $\tanh\left(\frac{2\pi d}{L}\right) \rightarrow 1$

For small d, $\tanh\left(\frac{2\pi d}{L}\right) \rightarrow \frac{2\pi d}{L}$

PROBLEM 2

Derive an expression of wave celerity in shallow water.

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L}$$

In shallow water (small *d*),

$$\boxed{\tanh\left(\frac{2\pi d}{L}\right) \to \frac{2\pi d}{L}}$$

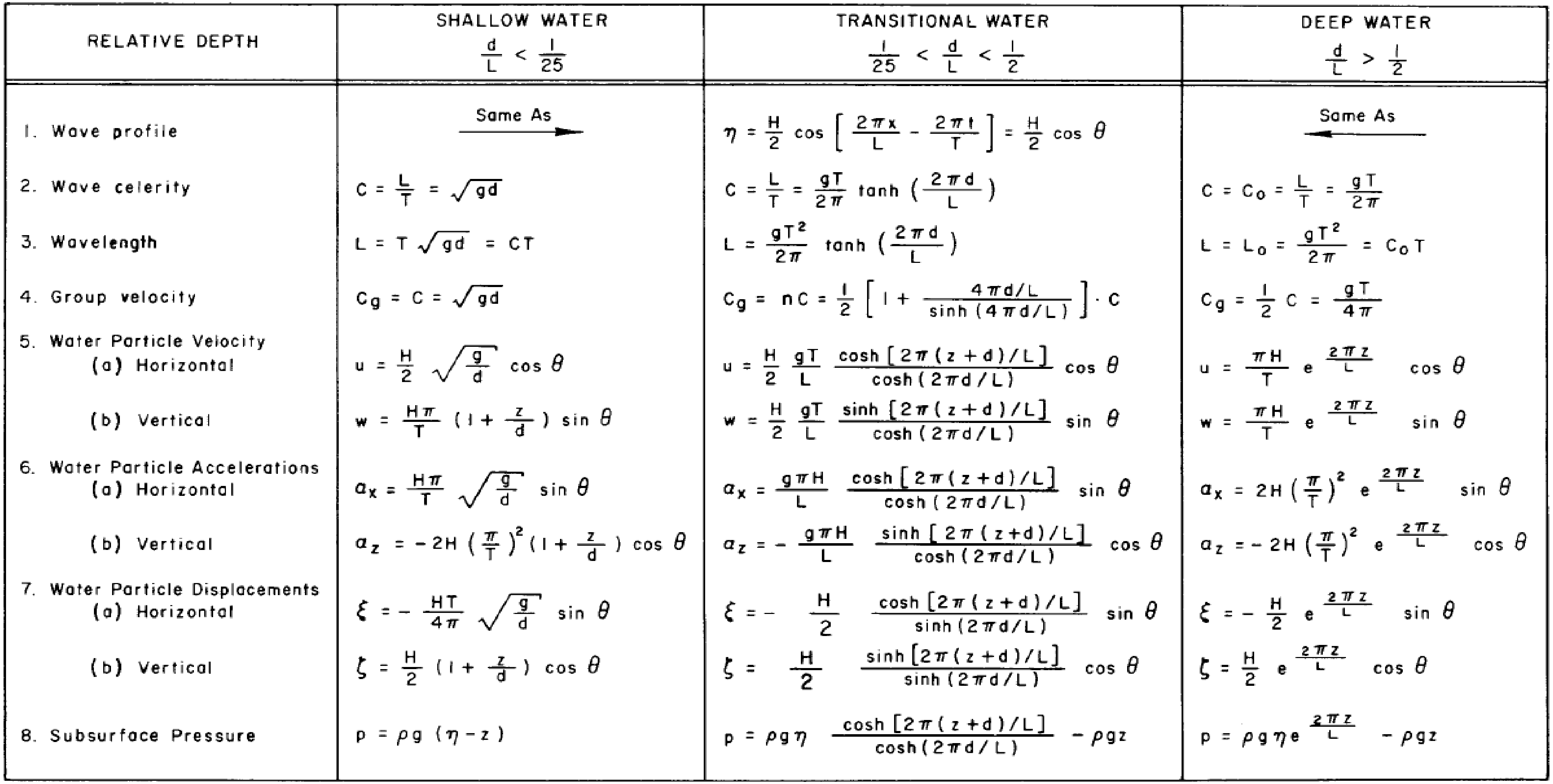
Celerity in shallow water ⟶

$$C = \frac{gT}{2\pi}\left(\frac{2\pi d}{L}\right)$$

$$C = \frac{gTd}{L} = \frac{gd}{C}$$

$$C = \sqrt{gd}$$

LINEAR WAVE THEORY - EQUATIONS

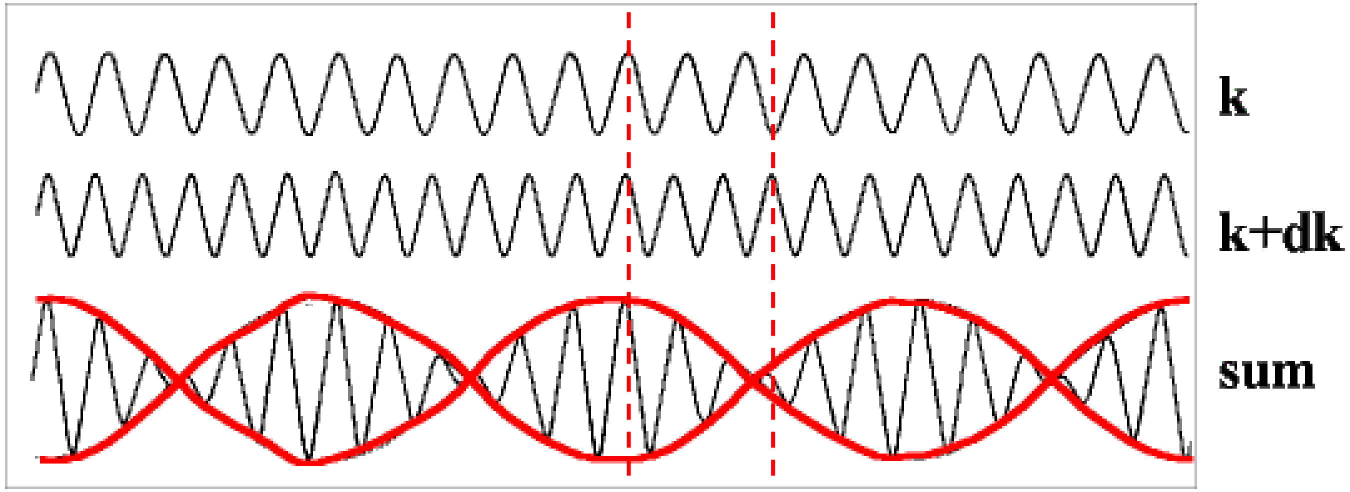


RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As $\longrightarrow$	$\eta = \frac{H}{2} \cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2} \cos\theta$	Same As $\longleftarrow$
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$L = L_o = \frac{gT^2}{2\pi} = C_o T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}} \cos\theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right) \sin\theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}} \sin\theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right) \cos\theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}} \sin\theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin\theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right) \cos\theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos\theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

WAVE GROUP

Two sinusoidal wave trains moving in the same direction with slightly different wavelengths and periods interact. The equation of the water surface is given by:

$$\eta = \eta_1 + \eta_2 = \frac{H}{2}\cos\left(\frac{2\pi x}{L_1} - \frac{2\pi t}{T_1}\right) + \frac{H}{2}\cos\left(\frac{2\pi x}{L_2} - \frac{2\pi t}{T_2}\right)$$



Waves of almost the same period interfere and tend to travel together in the same direction, forming wave groups.

WAVE GROUP

The speed of the wave group, C_G relates to velocity of propagation, C and the group velocity factor, n.

$$\boxed{C_G = nC}$$

Group velocity factor, n:

$$\boxed{n = \frac{1}{2}\left[1 + \frac{4\pi d / L}{\sinh(4\pi d / L)}\right]}$$

Deep water, $n = 0.5$; $C_G = 0.5C$

Shallow water, $n = 1$; $C_G = C$

Transitional water, $0.5 < n < 1$

Table C-1 (Shore Protection Manual, 1984)

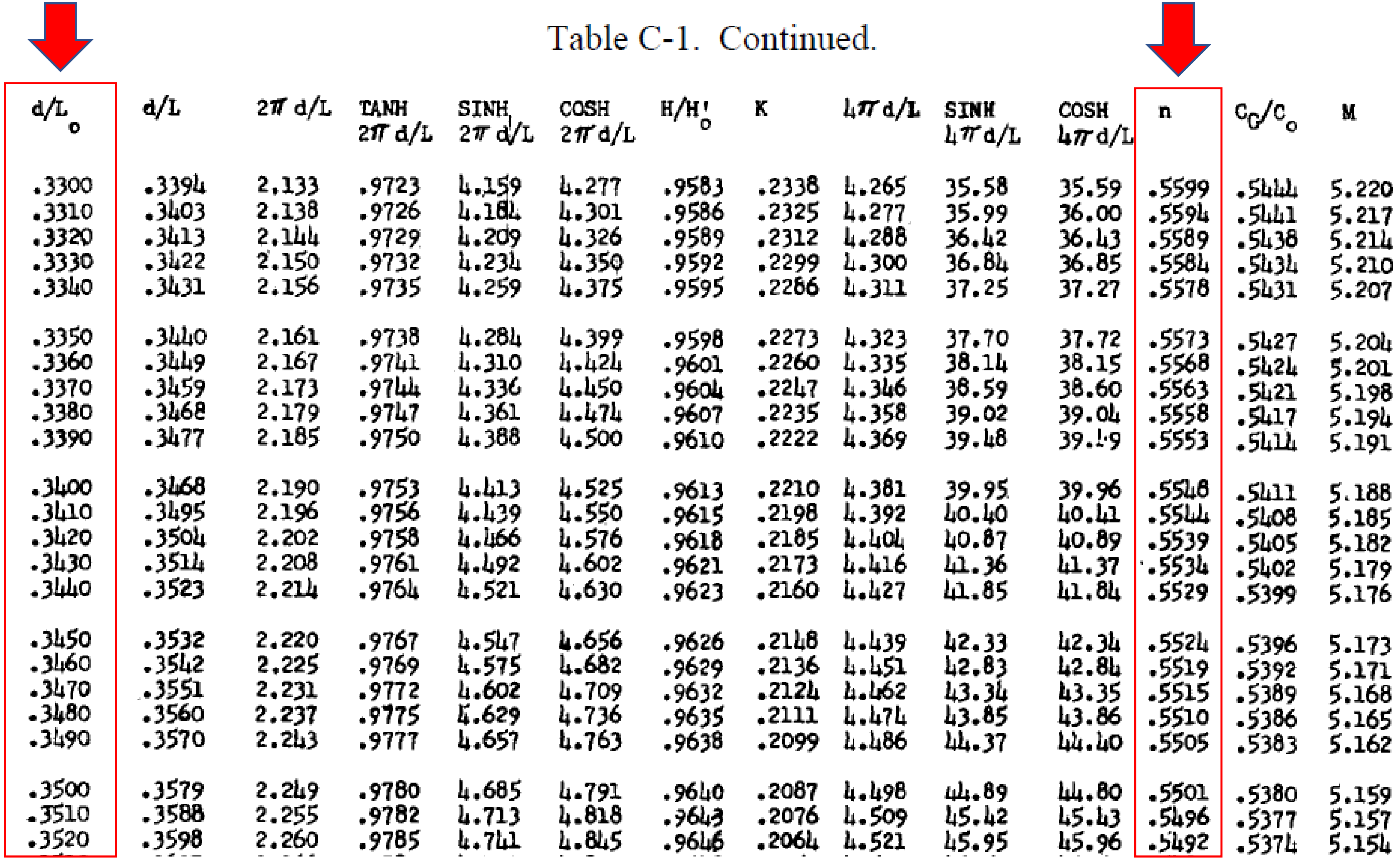


Table C-1. Continued.

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H'_o	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.3300	.3394	2.133	.9723	4.159	4.277	.9583	.2338	4.265	35.58	35.59	.5599	.5444	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.9586	.2325	4.277	35.99	36.00	.5594	.5441	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.9589	.2312	4.288	36.42	36.43	.5589	.5438	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.9592	.2299	4.300	36.84	36.85	.5584	.5434	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.9595	.2286	4.311	37.25	37.27	.5578	.5431	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.9598	.2273	4.323	37.70	37.72	.5573	.5427	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.9601	.2260	4.335	38.14	38.15	.5568	.5424	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.9604	.2247	4.346	38.59	38.60	.5563	.5421	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.9607	.2235	4.358	39.02	39.04	.5558	.5417	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.9610	.2222	4.369	39.48	39.49	.5553	.5414	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.9613	.2210	4.381	39.95	39.96	.5548	.5411	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.9615	.2198	4.392	40.40	40.41	.5544	.5408	5.185
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.3440	.3523	2.214	.9764	4.521	4.630	.9623	.2160	4.427	41.85	41.84	.5529	.5399	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.9626	.2148	4.439	42.33	42.34	.5524	.5396	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.9629	.2136	4.451	42.83	42.84	.5519	.5392	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.9632	.2124	4.462	43.34	43.35	.5515	.5389	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.9635	.2111	4.474	43.85	43.86	.5510	.5386	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.9638	.2099	4.486	44.37	44.40	.5505	.5383	5.162
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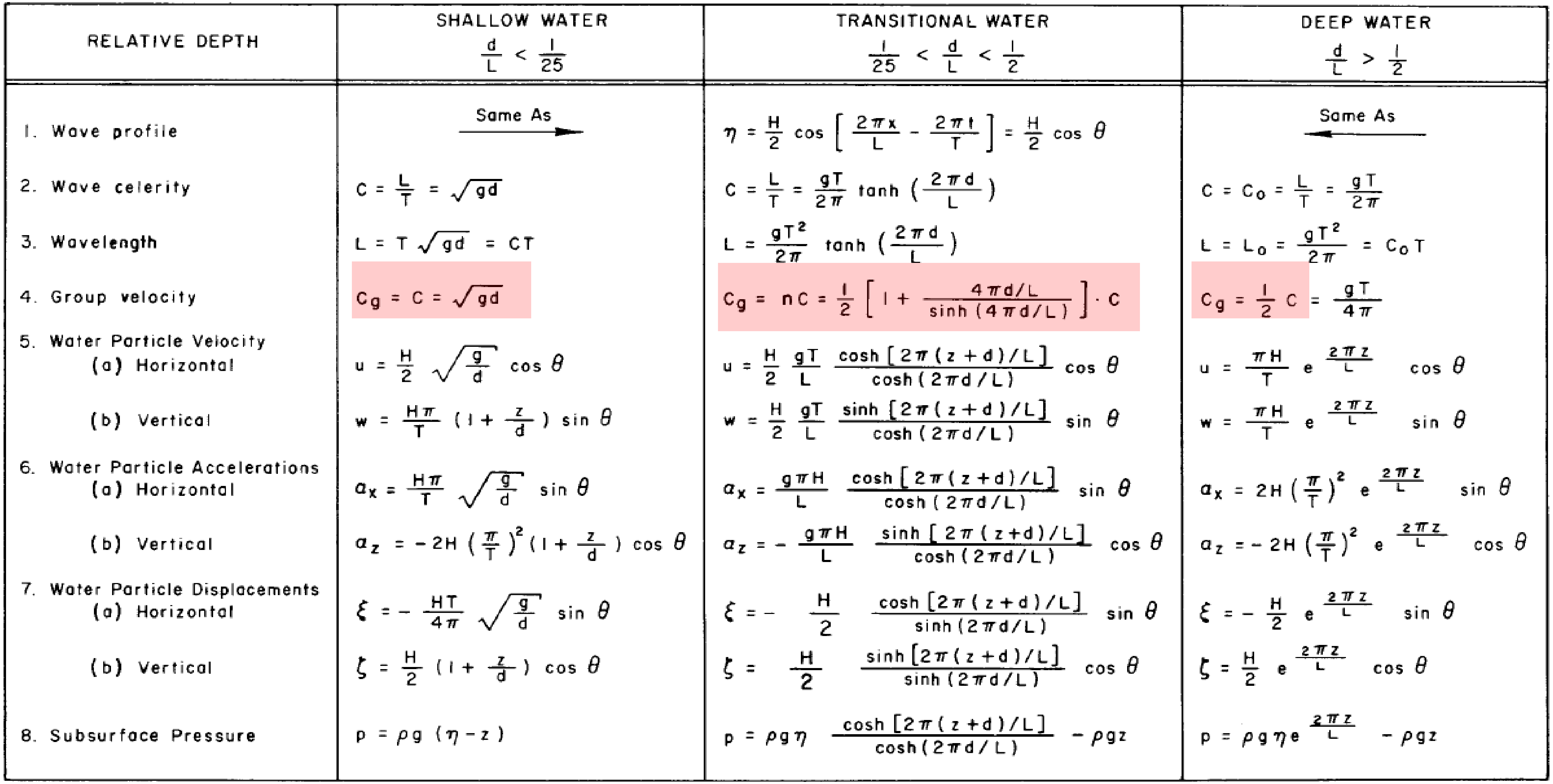
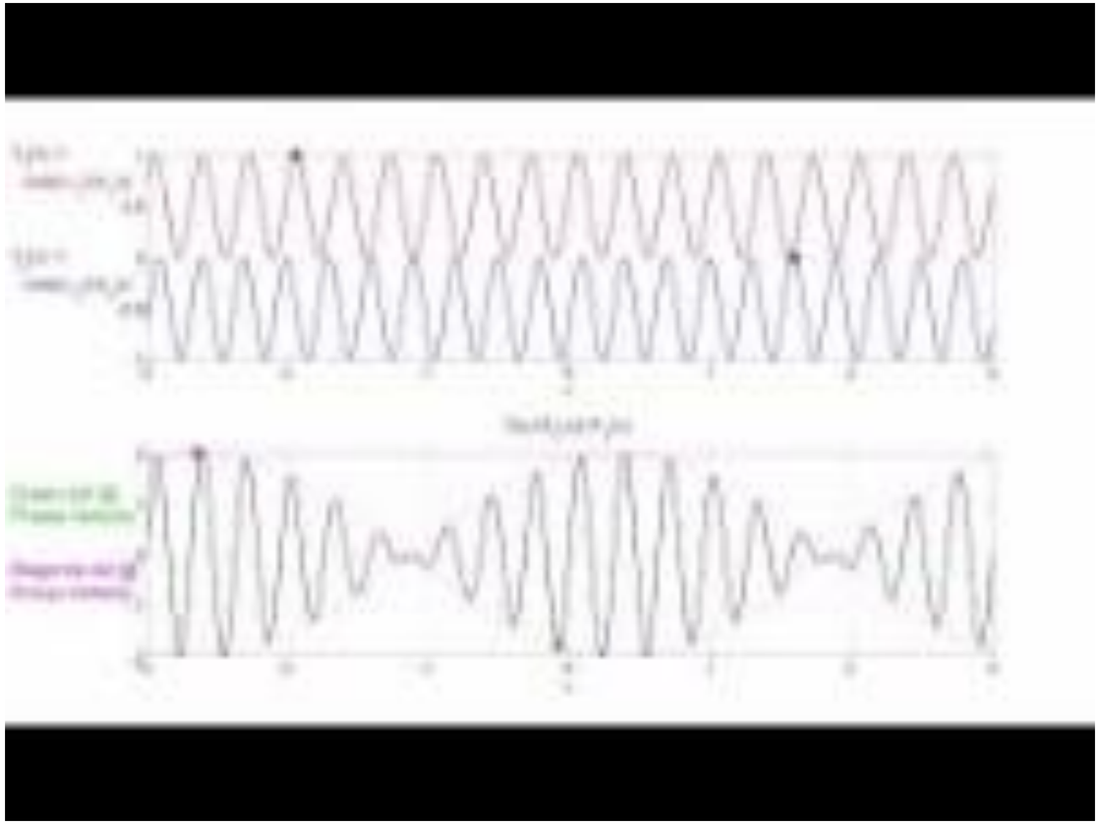
.3510	.3588	2.255	.9782	4.713	4.818	.9643	.2076	4.509	45.42	45.43	.5496	.5377	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.9646	.2064	4.521	45.95	45.96	.5492	.5374	5.154

LINEAR WAVE THEORY - EQUATIONS



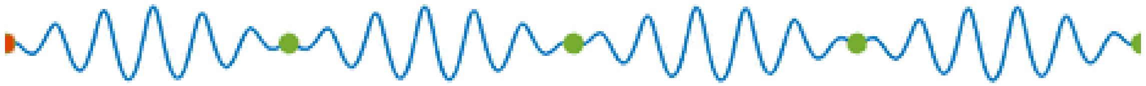
RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_o = \frac{gT^2}{2\pi} = C_o T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos \theta$
(b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin \theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$a_z = -2H \left(\frac{\pi}{T} \right)^2 \left(1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \cos \theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$\zeta = \frac{H}{2} \left(1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos \theta$
8. Subsurface Pressure	$p = \rho g (\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

PHASE VELOCITY VS. GROUP VELOCITY



https://youtu.be/tlM9vq-bepA

- The individual wave speed is the phase velocity or wave celerity, C.
- The group speed is termed the group velocity, C_G.
- The speed a group of waves or a wave train travels is generally not identical to the speed with which individual waves within the group travel.
- For waves propagating in deep or transitional water, $C > C_G$.
- For waves propagating in shallow water, $C \cong C_G$.



● Group velocity, C_G

■ Phase velocity, C

PROBLEM 3

A wave has a period of 10 s and a height of 2 m in deep water.

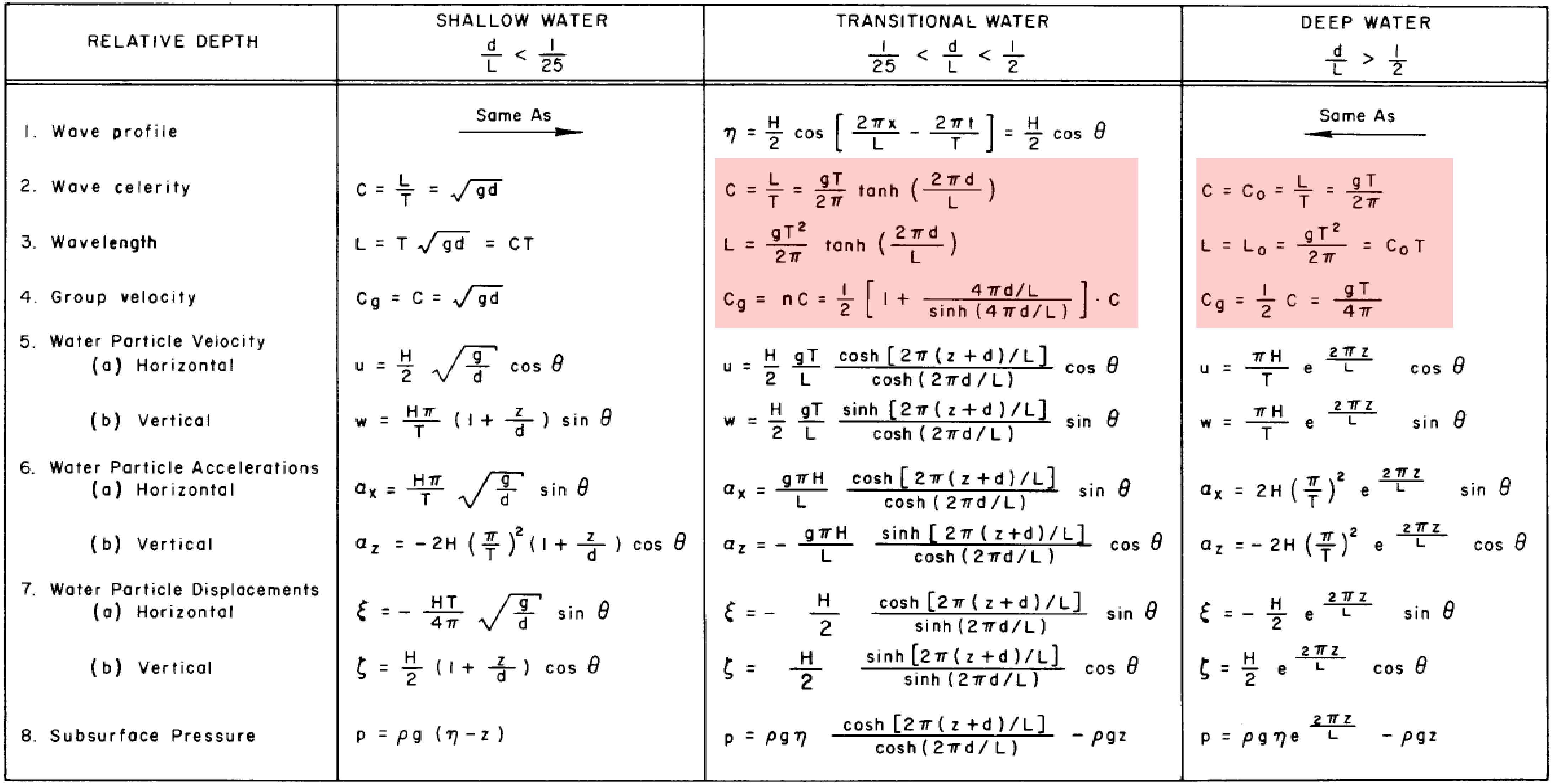
Determine

a. the wave celerity

b. the wavelength, and

c. the wave group velocity.

LINEAR WAVE THEORY - EQUATIONS



RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2} \cos\theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$L = L_o = \frac{gT^2}{2\pi} = C_o T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}} \cos\theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right) \sin\theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}} \sin\theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right) \cos\theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}} \sin\theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin\theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right) \cos\theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos\theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

PROBLEM 3

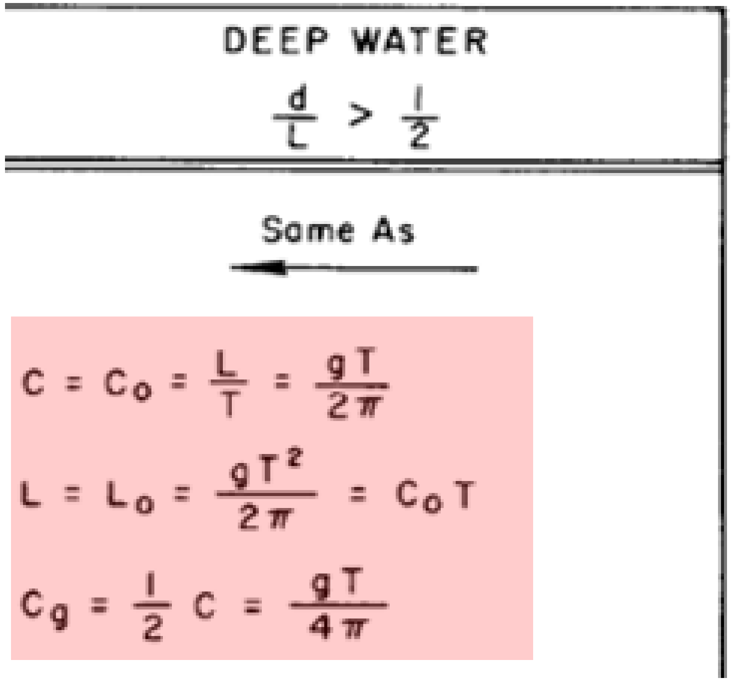
A wave has a period of **$T = 10$ s** and a height of **$H = 2$ m** in **deep water**.

Determine

a. the wave celerity, **C_o**

b. the wavelength **L_o**, and

c. the wave group velocity **$C_{g,o}$**.



DEEP WATER

$\frac{d}{L} > \frac{1}{2}$

Same As ←

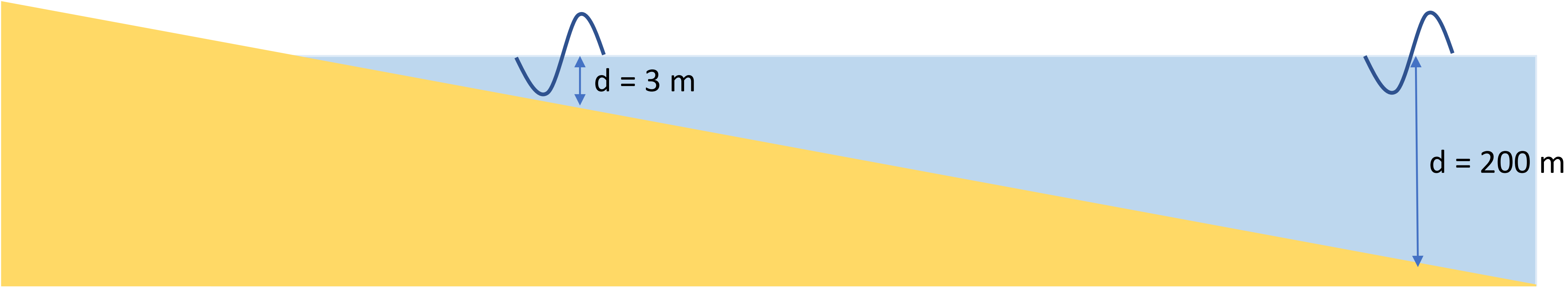
$$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$$

$$L = L_o = \frac{gT^2}{2\pi} = C_o T$$

$$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$$

PROBLEM 4

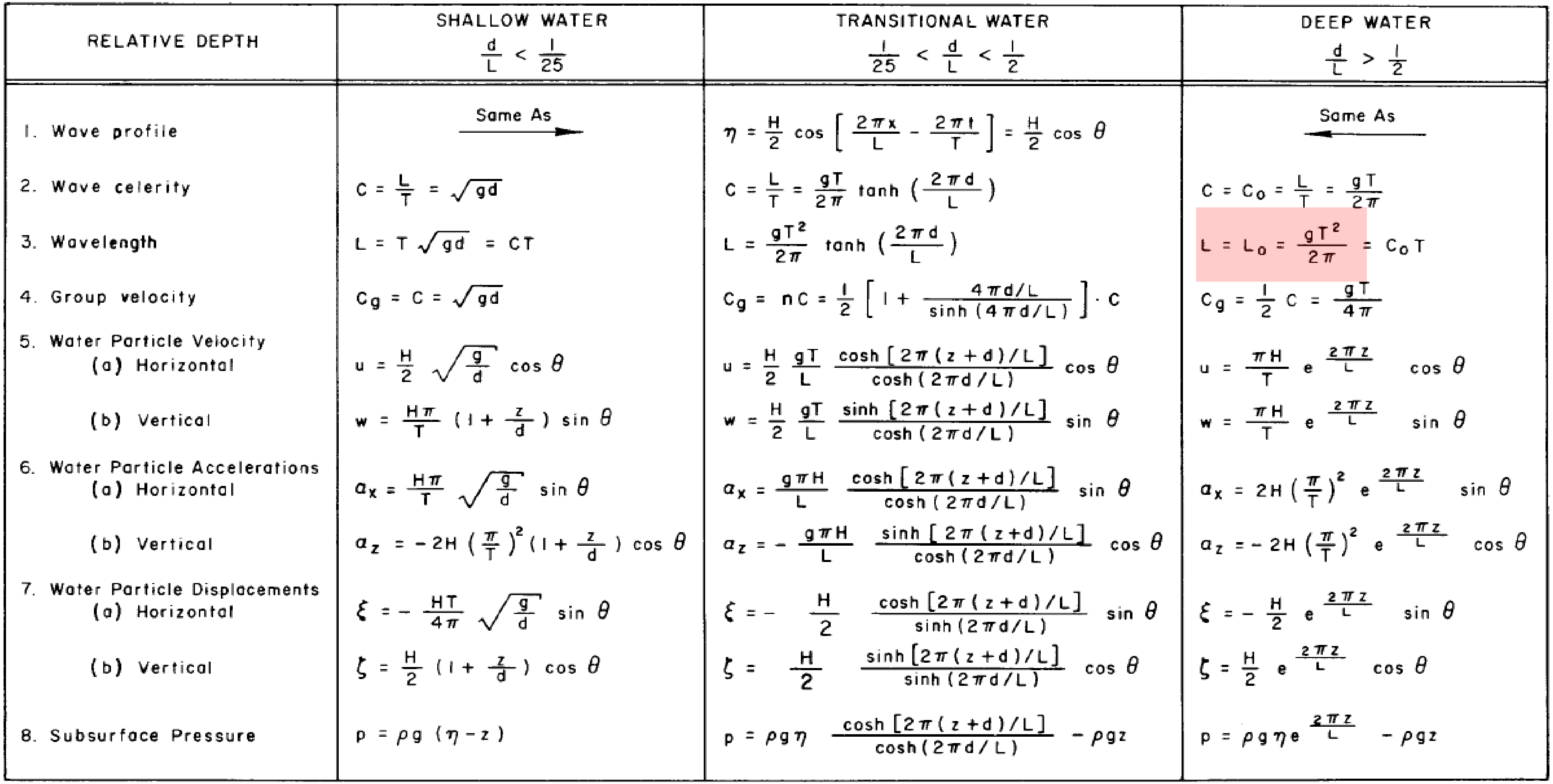
A wave with a period of 10 s is propagated shoreward over a uniformly sloping shelf from a depth 200 m to a depth of 3 m. Determine wave celerities and lengths corresponding to depths 200 m and 3 m.



T = 10 s
Water condition = ?
C = ?
L = ?

T = 10 s
Water condition = ?
C = ?
L = ?

LINEAR WAVE THEORY - EQUATIONS



RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2}\cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = L_o = \frac{gT^2}{2\pi} = C_o T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]\cdot C$	$C_g = \frac{1}{2}C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos\theta$	$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$u = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right)\sin\theta$	$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$w = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}}\sin\theta$	$a_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right)\cos\theta$	$a_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}}\sin\theta$	$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$	$\xi = -\frac{H}{2}e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right)\cos\theta$	$\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$	$\zeta = \frac{H}{2}e^{\frac{2\pi z}{L}}\cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g\eta\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho gz$	$p = \rho g\eta e^{\frac{2\pi z}{L}} - \rho gz$

SOLUTION

$$L_o = \frac{gT^2}{2\pi} = \frac{9.8}{2\pi}T^2 = 1.56T^2 \text{ m } (5.12T^2 \text{ ft})$$

$$L_o = 1.56T^2 = 1.56(10)^2 = 156 \text{ m } (512 \text{ ft})$$

For d = 200 meters

$$\frac{d}{L_o} = \frac{200}{156} = 1.2821$$

From Table C-1 it is seen that for values of

$$\frac{d}{L_o} > 1.0$$

$$\frac{d}{L_o} = \frac{d}{L}$$

therefore,

$$L = L_o = 156 \text{ m } (512 \text{ ft}) \left(\text{deepwater wave, since } \frac{d}{L} > \frac{1}{2}\right)$$

By equation (2-1)

$$C = \frac{L}{T} = \frac{156}{T}$$

$$C = \frac{156}{10} = 15.6 \text{ m/s } (51.2 \text{ ft/s})$$

For d = 3 meters

$$\frac{d}{L_o} = \frac{3}{156} = 0.0192$$

Entering Table C-1 with d/L_o it is found that,

$$\frac{d}{L} = 0.05641$$

hence,

$$L = \frac{3}{0.05641} = 53.2 \text{ m } (174 \text{ ft}) \left(\text{transitional depth, since } \frac{1}{25} < \frac{d}{L} < \frac{1}{2}\right)$$

$$C = \frac{L}{T} = \frac{53.2}{10} = 5.32 \text{ m/s } (17.4 \text{ ft/s})$$

Table C-1 (SPM, pp. C-17)

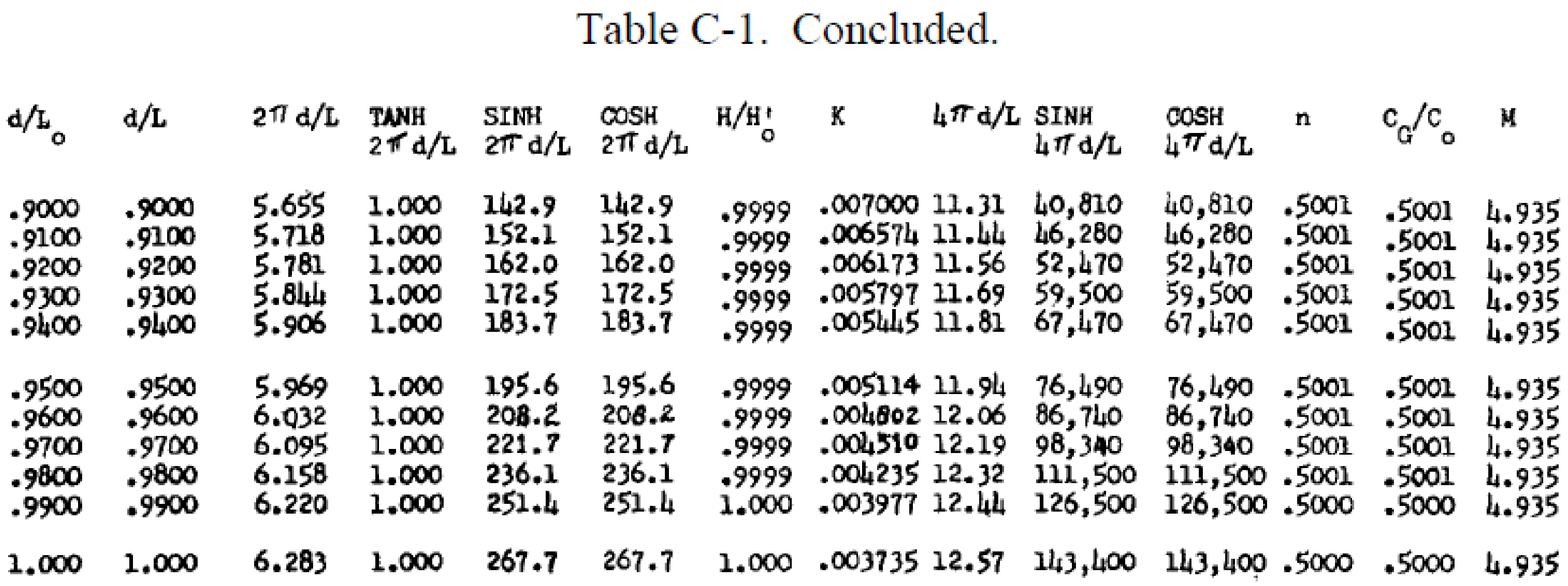


Table C-1. Concluded.

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H_o'	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.9000	.9000	5.655	1.000	142.9	142.9	.9999	.007000	11.31	40,810	40,810	.5001	.5001	4.935
.9100	.9100	5.718	1.000	152.1	152.1	.9999	.006574	11.44	46,280	46,280	.5001	.5001	4.935
.9200	.9200	5.781	1.000	162.0	162.0	.9999	.006173	11.56	52,470	52,470	.5001	.5001	4.935
.9300	.9300	5.844	1.000	172.5	172.5	.9999	.005797	11.69	59,500	59,500	.5001	.5001	4.935
.9400	.9400	5.906	1.000	183.7	183.7	.9999	.005445	11.81	67,470	67,470	.5001	.5001	4.935
.9500	.9500	5.969	1.000	195.6	195.6	.9999	.005114	11.94	76,490	76,490	.5001	.5001	4.935
.9600	.9600	6.032	1.000	208.2	208.2	.9999	.004802	12.06	86,740	86,740	.5001	.5001	4.935
.9700	.9700	6.095	1.000	221.7	221.7	.9999	.004510	12.19	98,340	98,340	.5001	.5001	4.935
.9800	.9800	6.158	1.000	236.1	236.1	.9999	.004235	12.32	111,500	111,500	.5001	.5001	4.935
.9900	.9900	6.220	1.000	251.4	251.4	1.000	.003977	12.44	126,500	126,500	.5000	.5000	4.935
1.000	1.000	6.283	1.000	267.7	267.7	1.000	.003735	12.57	143,400	143,400	.5000	.5000	4.935

$d/L_o = 1.2821 = d/L$

after Wiegel, R. L., "Oscillatory Waves," U.S. Army, Beach Erosion Board, Bulletin, Special Issue No. 1, July 1948.

Table C-1 (SPM, pp. C-6)

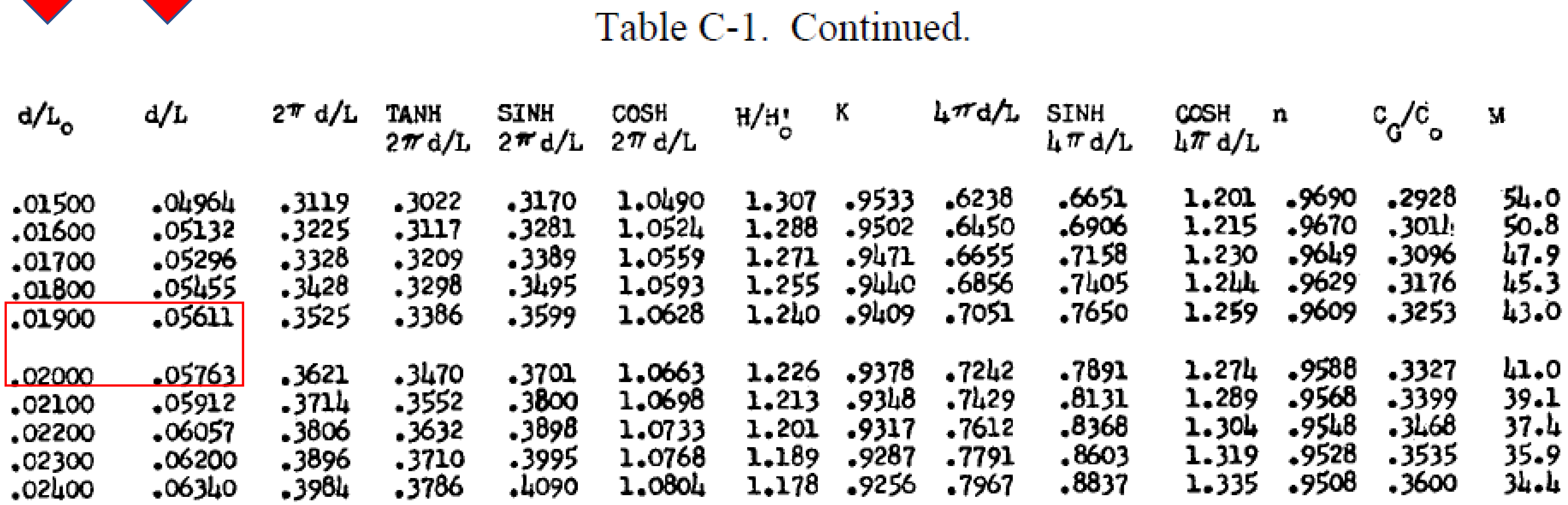


Table C-1. Continued.

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H_o'	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.01500	.04964	.3119	.3022	.3170	1.0490	1.307	.9533	.6238	.6651	1.201	.9690	.2928	54.0
.01600	.05132	.3225	.3117	.3281	1.0524	1.288	.9502	.6450	.6906	1.215	.9670	.3014	50.8
.01700	.05296	.3328	.3209	.3389	1.0559	1.271	.9471	.6655	.7158	1.230	.9649	.3096	47.9
.01800	.05455	.3428	.3298	.3495	1.0593	1.255	.9440	.6856	.7405	1.244	.9629	.3176	45.3
.01900	.05611	.3525	.3386	.3599	1.0628	1.240	.9409	.7051	.7650	1.259	.9609	.3253	43.0
.02000	.05763	.3621	.3470	.3701	1.0663	1.226	.9378	.7242	.7891	1.274	.9588	.3327	41.0
.02100	.05912	.3714	.3552	.3800	1.0698	1.213	.9348	.7429	.8131	1.289	.9568	.3399	39.1
.02200	.06057	.3806	.3632	.3898	1.0733	1.201	.9317	.7612	.8368	1.304	.9548	.3468	37.4
.02300	.06200	.3896	.3710	.3995	1.0768	1.189	.9287	.7791	.8603	1.319	.9528	.3535	35.9
.02400	.06340	.3984	.3786	.4090	1.0804	1.178	.9256	.7967	.8837	1.335	.9508	.3600	34.4

$d/L_o = 0.0192$ →

Calibration: $$d/L = 0.05611 + \left(\frac{0.05763 - 0.05611}{0.020 - 0.019} \times (0.0192 - 0.0190)\right) = 0.05641$$

SOLUTION

For d = 3 meters

An approximate value of L can also be found by using equation (2-4b)

$$L \approx \frac{gT^2}{2\pi}\sqrt{\tanh\left(\frac{4\pi^2}{T^2}\,\frac{d}{g}\right)}$$

which can be written in terms of L_o as

$$L \approx L_o \sqrt{\tanh\left(\frac{2\pi d}{L_o}\right)}$$

therefore,

$$L \approx 156 \sqrt{\tanh \frac{2\pi(3)}{156}}$$

$$L \approx 156 \sqrt{\tanh(0.1208)}$$

$$L \approx 156 \sqrt{0.1202} = 54.1 \text{ m } (177.5 \text{ ft})$$

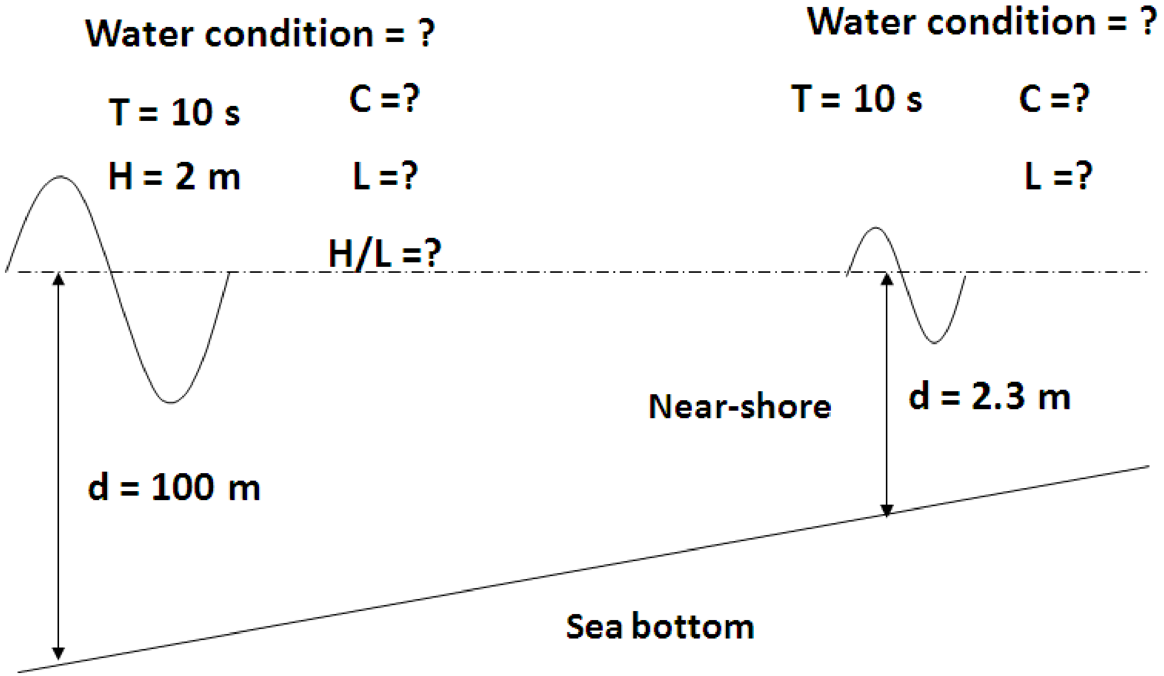
which compares with L = 53.3 meters obtained using Table C-1. The error in this case is 1.5 percent. Note that Plate C-1 could also have been used to determine d/L.

PROBLEM 5

A wave in water 100 m deep has a period of 10 s and a height of 2 m.

(a) Determine the water condition, wave celerity, length, and steepness.

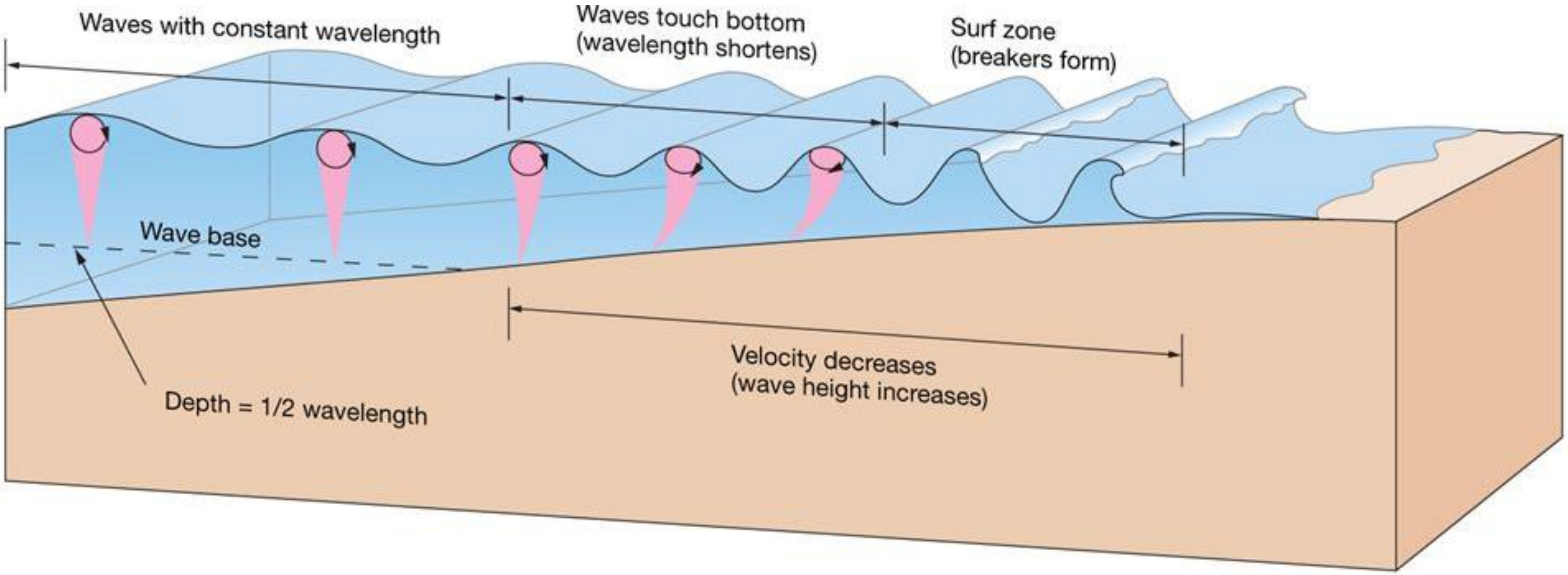
(b) Calculate the wavelength and celerity when it has propagated into a near-shore depth of 2.3 m.



SOLUTION

1. Find L_o. $$L_o = \frac{gT^2}{2\pi}$$
2. d/L_o = ?
3. Obtain the corresponding d/L from Table C-1 (SPM) based on d/L_o. Calibration of d/L up to 4 decimal points may be needed.
4. Identify the water condition (i.e. deepwater/ transitional/ shallow) based on the d/L value.
5. Use equations to determine other wave properties.

CHANGE OF WATER PARTICLE DISPLACEMENT



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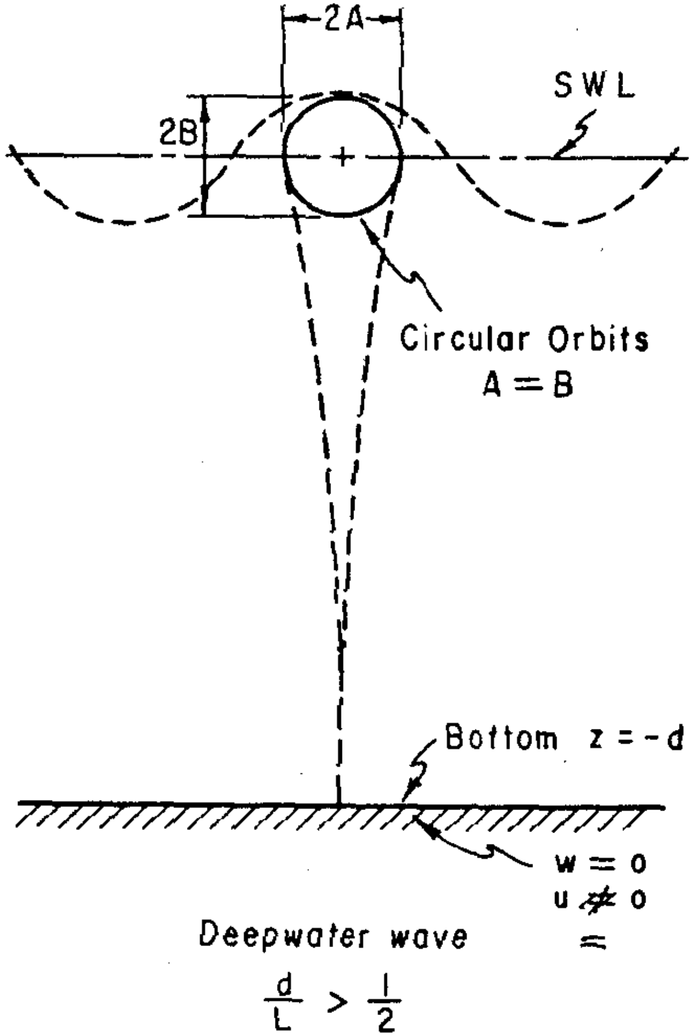
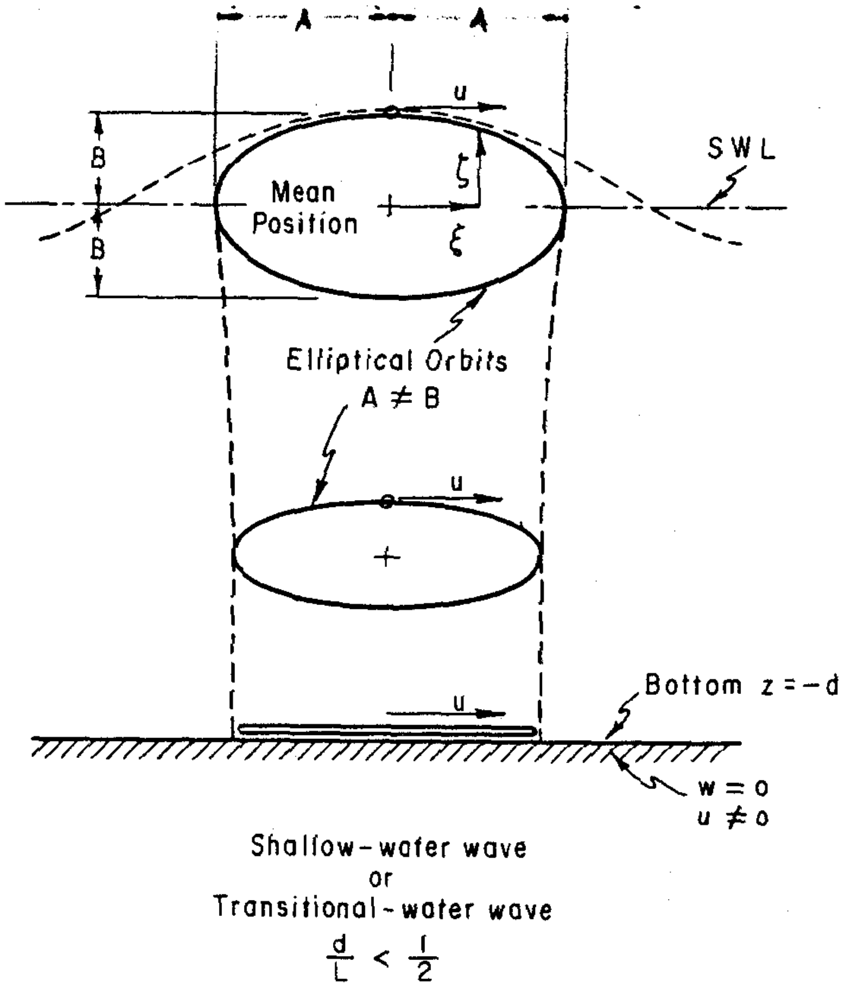
WATER PARTICLE DISPLACEMENT IN DEEP WATERS



- Water particle orbits are circular.
- The amplitude of water particle displacement decreases exponentially with depth.
- Orbital movement stops at $z = L_o/2$
- Vertical & horizontal water particle displacements are the same.
- Maximum horizontal & vertical water particle displacement from the origin:

$$\boxed{A = B = \frac{H}{2} e^{\frac{2\pi z}{L_o}}}$$

WATER PARTICLE DISPLACEMENT IN TRANSITIONAL/SHALLOW WATERS

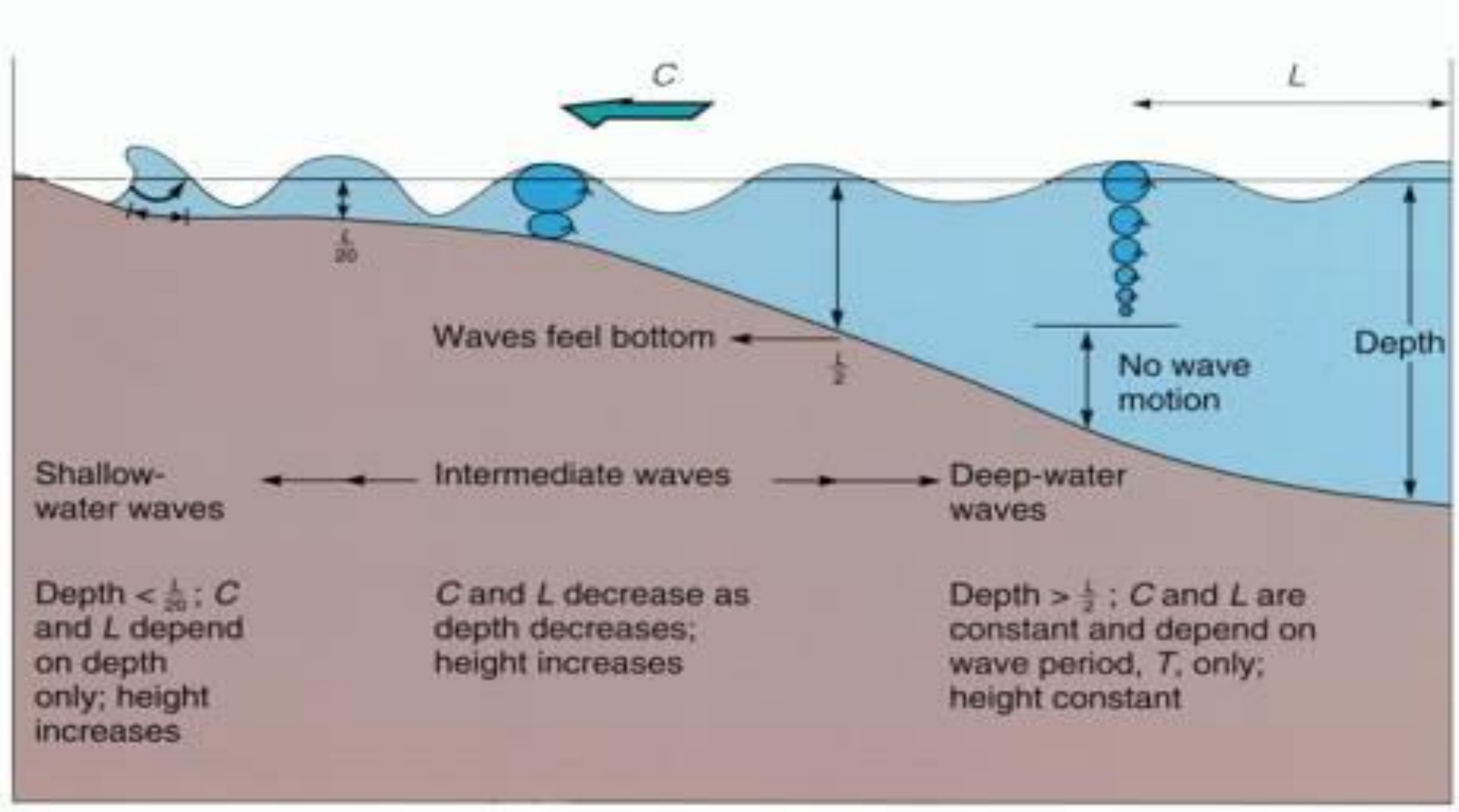


- Water particle orbits are elliptical.
- The shallower the water, the flatter the ellipse.
- The amplitude of water particle displacement decreases exponentially with depth.
- At the bottom ($z = -d$), the particles follow a reversing horizontal path.
- Vertical displacement :
  - (a) At bottom, $\zeta = 0$
  - (b) At water surface, $\zeta = H/2$

LINEAR WAVE THEORY - EQUATIONS

RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2}\cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	Same As ←
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]\cdot C$	$C_g = \frac{1}{2}C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos\theta$	$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$u = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right)\sin\theta$	$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$w = \frac{\pi H}{T}e^{\frac{2\pi z}{L}}\sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}}\sin\theta$	$a_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right)\cos\theta$	$a_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}}\sin\theta$	$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$	$\xi = -\frac{H}{2}e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right)\cos\theta$	$\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$	$\zeta = \frac{H}{2}e^{\frac{2\pi z}{L}}\cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g\eta\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g\eta e^{\frac{2\pi z}{L}} - \rho g z$

WATER PARTICLE DISPLACEMENTS AT DIFFERENT WATER CONDITIONS



Depth < $\frac{L}{20}$; C and L depend on depth only; height increases

C and L decrease as depth decreases; height increases

Depth > $\frac{L}{2}$; C and L are constant and depend on wave period, T, only; height constant

WATER PARTICLE DISPLACEMENT

Horizontal WP Displacement, ξ:

$$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$$

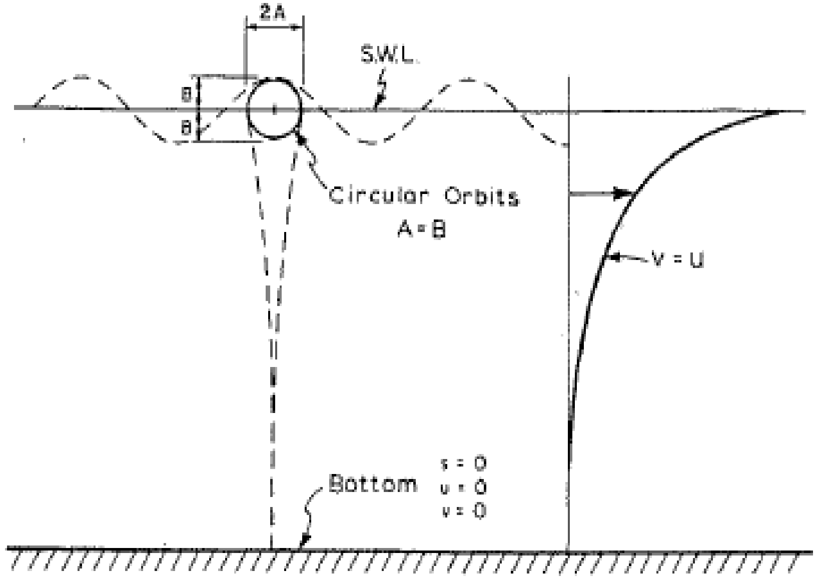
Vertical WP Displacement, ζ:

$$\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$$

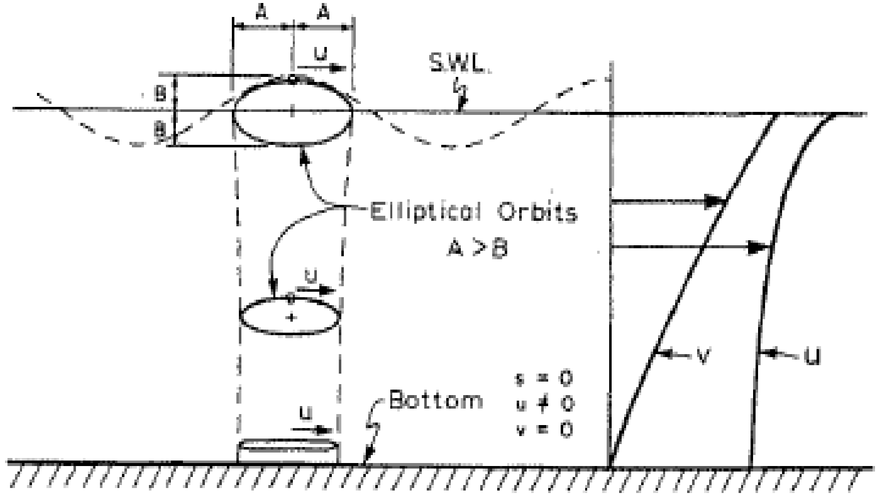
Where:

$$\theta = kx - \varpi t = \frac{2\pi x}{L} - \frac{2\pi t}{T}$$

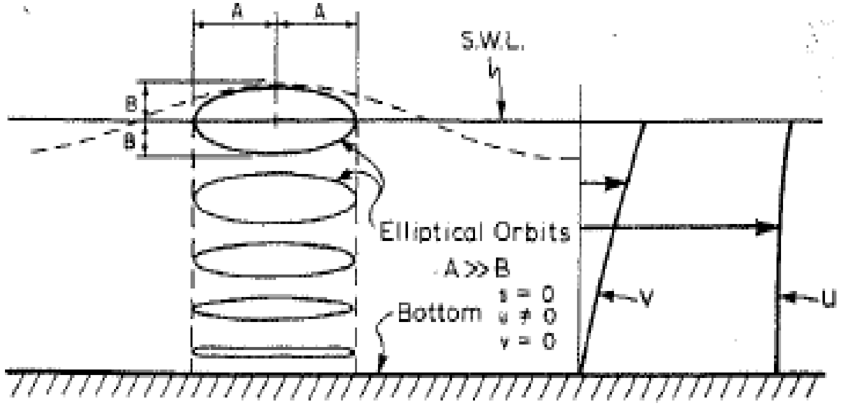
WATER PARTICLE VELOCITY



(a) DEEPWATER $\frac{d}{L} > \frac{1}{2}$



(b) INTERMEDIATE DEPTH $\frac{1}{20} < \frac{d}{L} < \frac{1}{2}$



(c) SHALLOW WATER $\frac{d}{L} > \frac{1}{20}$

WATER PARTICLE VELOCITIES

Horizontal WP Velocity, *u*:

$$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$$

- Maximum positive-*u* occurs when $\theta = 0, 2\pi$, etc.
- Maximum negative-*u* occurs when $\theta = \pi, 3\pi$, etc.

Vertical WP Velocity, *w*:

$$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$$

- Maximum positive-*w* occurs when $\theta = \pi/2, 5\pi/2$, etc.
- Maximum negative-*w* occurs when $\theta = 3\pi/2, 7\pi/2$, etc.

WATER PARTICLE ACCELERATIONS

Horizontal WP Acceleration, a_x:

$$\alpha_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta = \frac{\partial u}{\partial t}$$

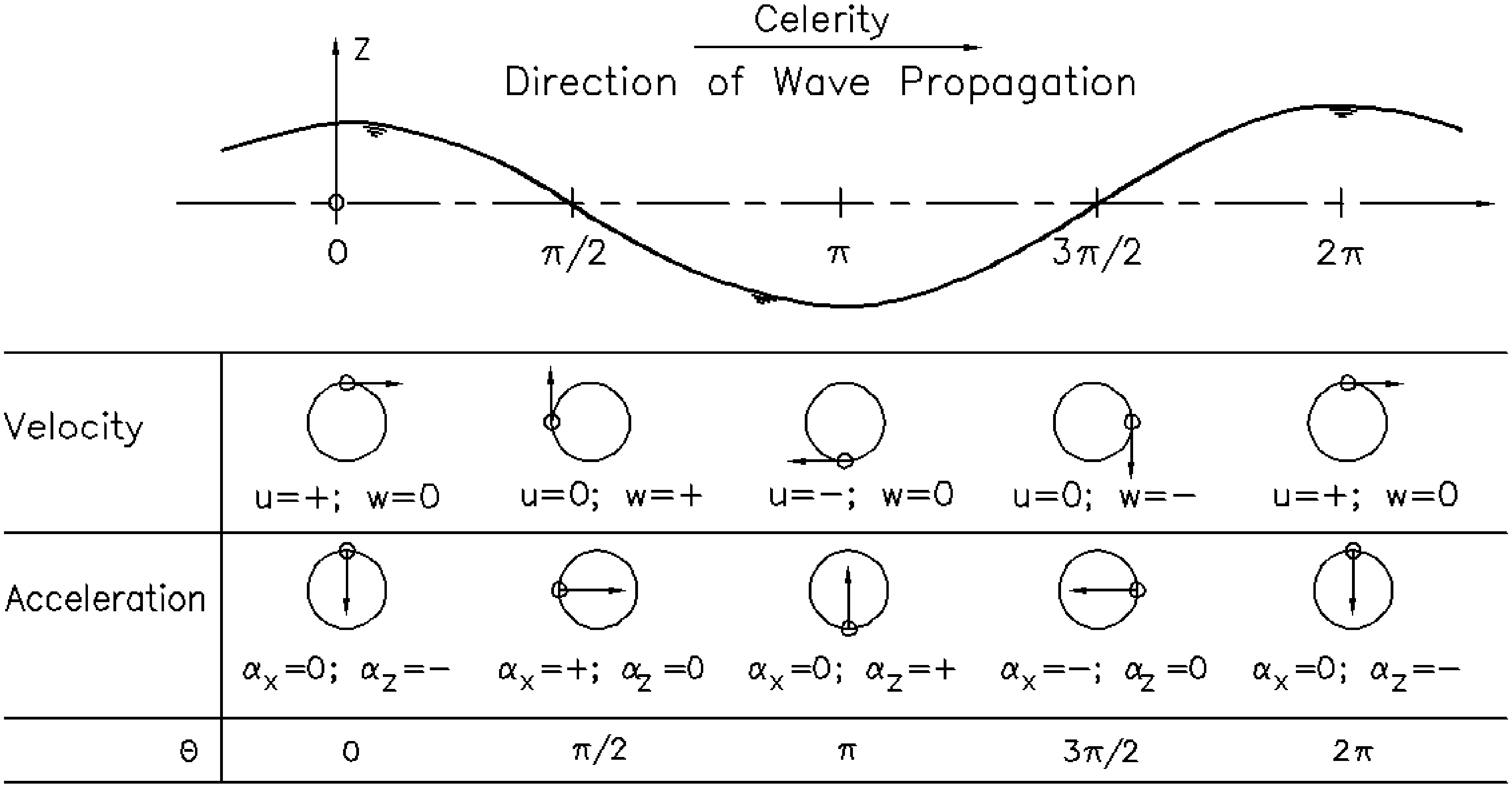
- Maximum positive-*w* occurs when $\theta = \pi/2, 5\pi/2$, etc.
- Maximum negative-*w* occurs when $\theta = 3\pi/2, 7\pi/2$, etc.

Vertical WP Acceleration, a_z :

$$\alpha_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta = \frac{\partial w}{\partial t}$$

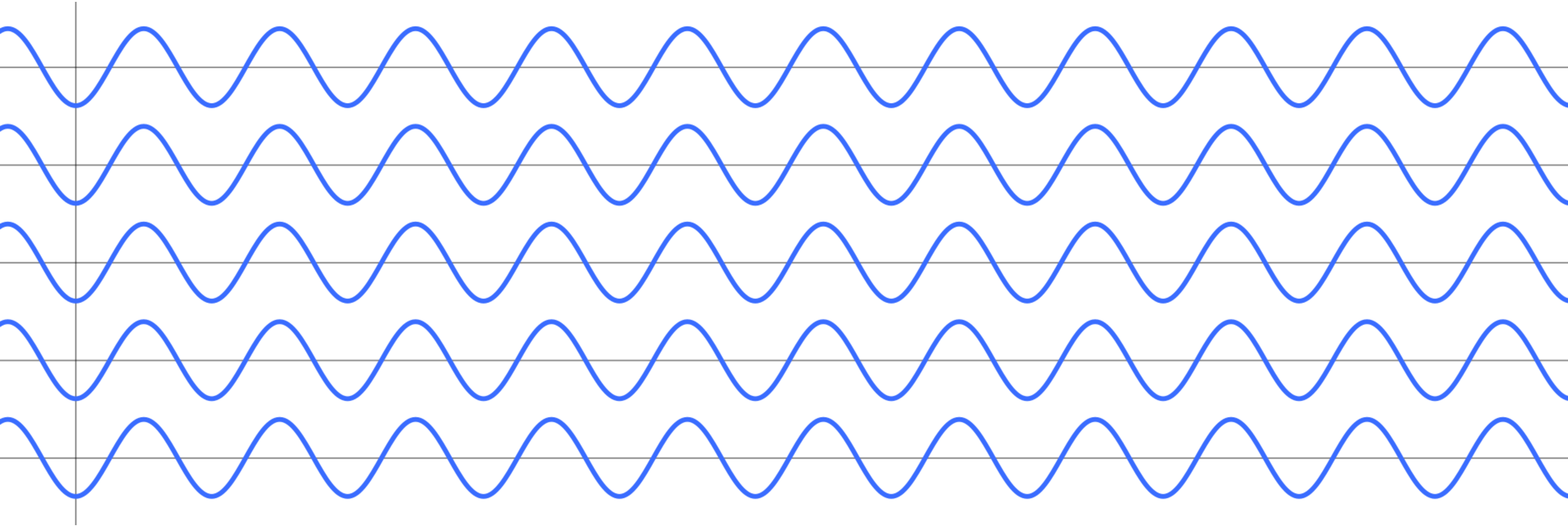
- Maximum positive-*u* occurs when $\theta = 0, 2\pi$, etc.
- Maximum negative-*u* occurs when $\theta = \pi, 3\pi$, etc.

OSCILLATORY FLUID MOTION

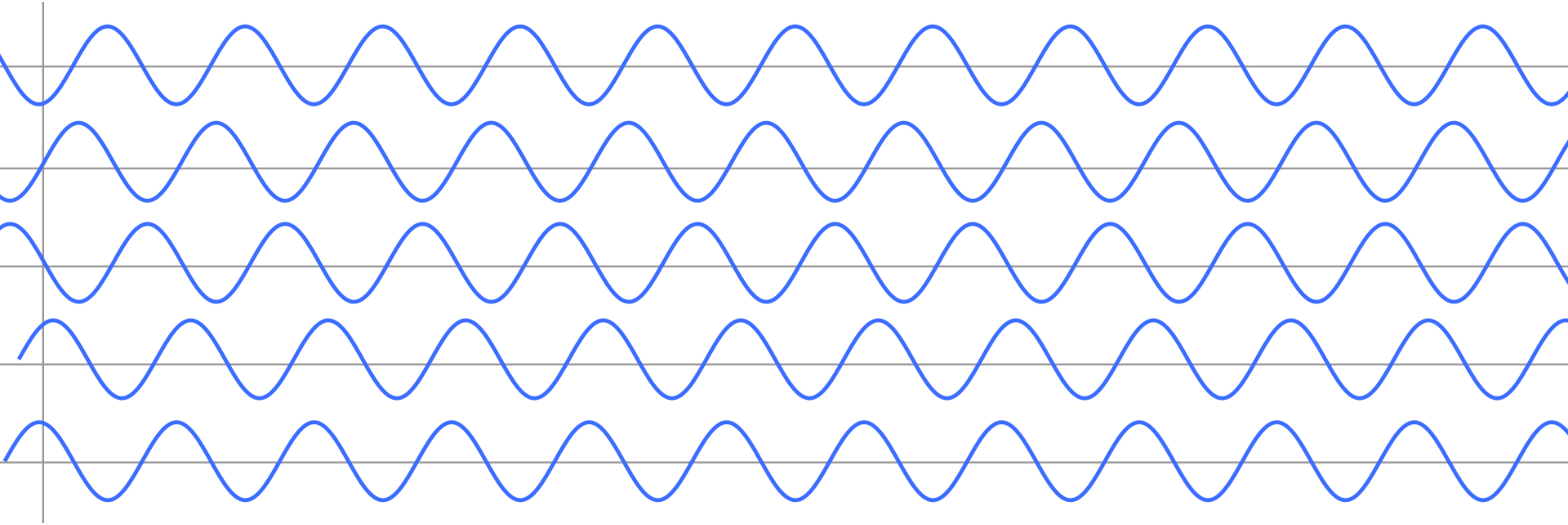


Velocity	u=+; w=0	u=0; w=+	u=−; w=0	u=0; w=−	u=+; w=0
Acceleration	$\alpha_x=0;\ \alpha_z=-$	$\alpha_x=+;\ \alpha_z=0$	$\alpha_x=0;\ \alpha_z=+$	$\alpha_x=-;\ \alpha_z=0$	$\alpha_x=0;\ \alpha_z=-$
θ	0	π/2	π	3π/2	2π

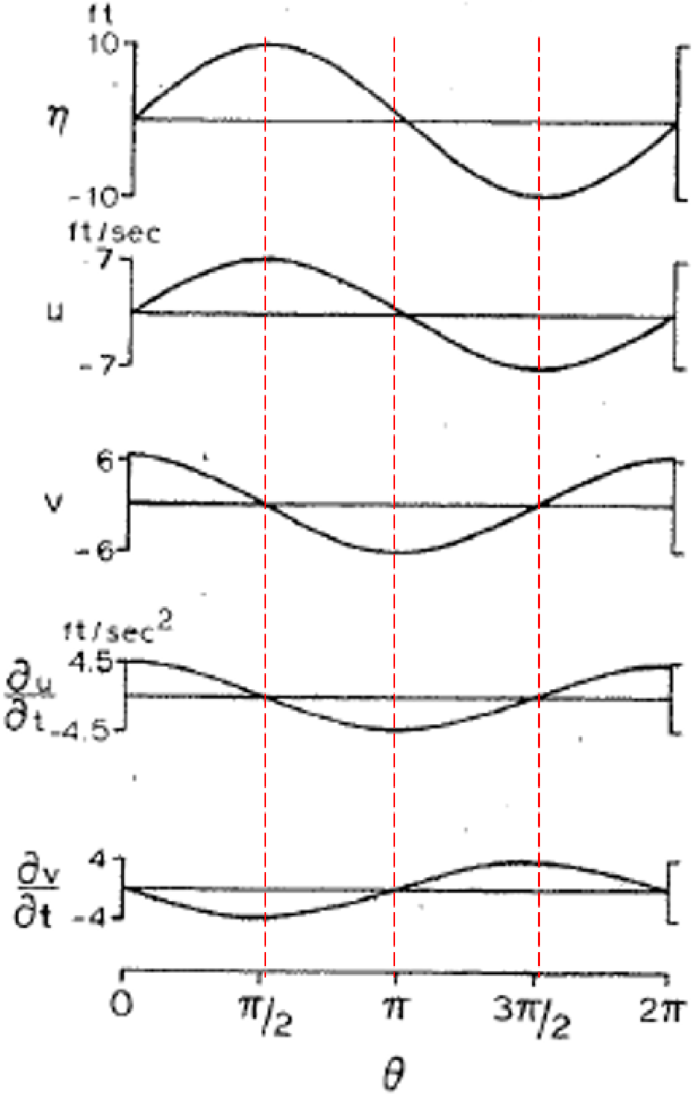
IN-PHASE WAVES



OUT-OF-PHASE WAVES



WAVE PROFILE AND THE KINEMATIC-DYNAMIC PROPERTIES



Wave profile:

Horizontal WP velocity:

In phase with the wave profile

Vertical WP velocity:

90° out of phase with the wave profile

Horizontal WP acceleration:

90° out of phase with the wave profile

Vertical WP acceleration:

180° out of phase with the wave profile

PROBLEM 6

A wave with a period of 8 s in a water depth of 15 m and a height of 5.5 m. Determine the horizontal and vertical velocities and accelerations at an elevation 5 m below the SWL when $\theta = 60°$.

SOLUTION

$$L_o = 1.56T^2 \text{ m} = (1.56)(8)^2 = 99.8 \text{ m } (327 \text{ ft})$$

$$\frac{d}{L_o} = \frac{15}{99.8} = 0.1503$$

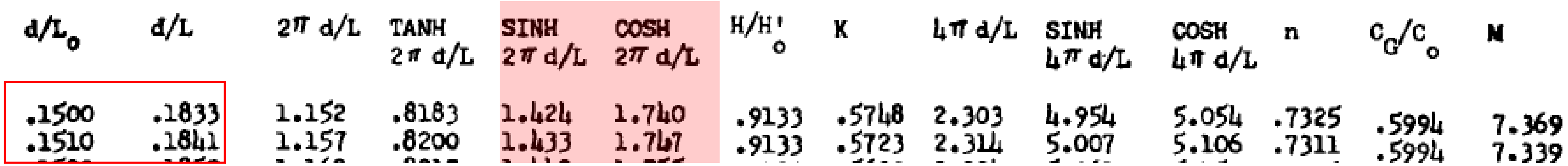
From Table C-1 in Appendix C for a value of

$$\frac{d}{L_o} = 0.1503$$

$$\frac{d}{L} \approx 0.1835; \quad \cosh \frac{2\pi d}{L} = 1.742$$

Table C-1 (SPM, pp. C-6)

$d/L_o = 0.1503$ →



d/L_o	d/L	$2\pi\, d/L$	TANH $2\pi\, d/L$	SINH $2\pi\, d/L$	COSH $2\pi\, d/L$	H/H_o'	K	$4\pi\, d/L$	SINH $4\pi\, d/L$	COSH $4\pi\, d/L$	n	C_G/C_o	M
.1500	.1833	1.152	.8183	1.424	1.740	.9133	.5748	2.303	4.954	5.054	.7325	.5994	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.9133	.5723	2.314	5.007	5.106	.7311	.5994	7.339

SOLUTION

Horizontal WP Velocity, *u*:

$$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$$

Vertical WP Velocity, *w*:

$$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$$

Horizontal WP Acceleration, a_x:

$$\alpha_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta = \frac{\partial u}{\partial t}$$

Vertical WP Acceleration, a_z:

$$\alpha_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta = \frac{\partial w}{\partial t}$$

hence,

$$L = \frac{15}{0.1835} = 81.7 \text{ m } (268 \text{ ft})$$

Evaluation of the constant terms in equations (2-13) to (2-16) gives

WP velocities:

$$\frac{HgT}{2L}\,\frac{1}{\cosh(2\pi d/L)} = \frac{5.5\ (9.8)(8)}{2\ (81.7)}\,\frac{1}{1.742} = 1.515$$

WP accelerations:

$$\frac{Hg\pi}{L}\,\frac{1}{\cosh(2\pi d/L)} = \frac{5.5\ (9.8)(3.1416)}{81.7}\,\frac{1}{1.742} = 1.190$$

Substitution into equation (2-13) gives

d + z = 15 + (-5)

$$u = 1.515 \cosh\left[\frac{2\pi(15-5)}{81.7}\right][\cos 60°] = 1.515\ [\cosh(0.7691)]\ (0.500)$$

$$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$$

From Table C-1 find

$$\frac{2\pi d}{L} = 0.7691$$

and by interpolation

cosh (0.7691) = 1.3106

and

sinh (0.7691) = 0.8472

SOLUTION

Horizontal WP Velocity, u:

$$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$$

Vertical WP Velocity, w:

$$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$$

Horizontal WP Acceleration, a_x:

$$\alpha_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta = \frac{\partial u}{\partial t}$$

Vertical WP Acceleration, a_z:

$$\alpha_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta = \frac{\partial w}{\partial t}$$

cosh (0.7691) = 1.3106

and

sinh (0.7691) = 0.8472

Therefore,

$u =$ 1.515 (1.3106) (0.500) = 0.99 m/s (3.26 ft/s) →

$w =$ 1.515 (0.8472) (0.866) = 1.11 m/s (3.65 ft/s) ↑

$\alpha_x =$ 1.190 (1.3106) (0.866) = 1.35 m/s^2 (4.43 ft/s^2) →

$\alpha_z =$ -1.190 (0.8472) (0.500) = -0.50 m/s^2 (1.65 ft/s^2) ↓

Problem 7

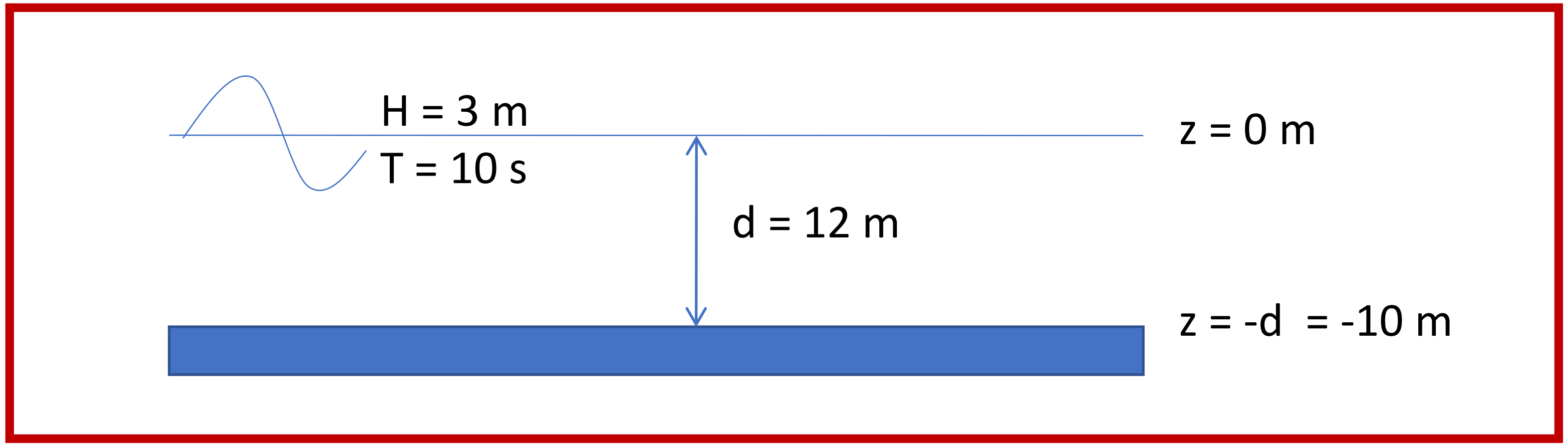
GIVEN: A wave in a depth d = 12 meters (39.4 feet), height H = 3 meters (9.8 feet), and a period T = 10 seconds. The corresponding deepwater wave height is H_o = 3.13 meters (10.27 feet).

FIND:

(a) The horizontal and vertical displacement of a water particle from its mean position when z = 0, and when z = -d.

(b) The maximum water particle displacement at an elevation z = 7.5 meters (-24.6 feet) when the wave is in infinitely deep water.

(c) For the deepwater conditions of (b) above, show that the particle displacements are small relative to the wave height when z = $-L_o/2$.



SOLUTION:

(a) $$L_o = 1.56T^2 = 1.56(10)^2 = 156 \text{ m } (512 \text{ ft})$$

$$\frac{d}{L_o} = \frac{12}{156} = 0.0769$$

From Appendix c, Table C-1

$$\sinh\left(\frac{2\pi d}{L}\right) = 0.8306$$

$$\tanh\left(\frac{2\pi d}{L}\right) = 0.6389$$

Water Particle Displacement at z = 0 m

Horizontal WP Displacement, ξ:

$$\xi = A = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta = -\frac{H}{2}\frac{\cosh[2\pi(0+d)/L]}{\sinh(2\pi d/L)}\sin\theta$$

$$\xi = A = -\frac{H}{2}\frac{1}{\dfrac{\sinh(2\pi d/L)}{\cosh(2\pi d/L)}}\sin\theta = -\frac{H}{2}\frac{1}{\tanh(2\pi d/L)}\sin\theta$$

Vertical WP Displacement, ζ:

$$\zeta = B = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta = \frac{H}{2}\frac{\sinh[2\pi(0+d)/L]}{\sinh(2\pi d/L)}\cos\theta$$

$$\zeta = B = \frac{H}{2}\cos\theta$$

Water Particle Displacement at *z* = 0 m

When $\boxed{z = 0,}$ equation (2-22) reduces to

$$A = \frac{H}{2} \frac{1}{\tanh(2\pi d/L)}$$

and equation (2-23) reduces to

$$B = \frac{H}{2}$$

Thus

$$A = \frac{3}{2} \frac{1}{(0.6389)} = 2.35 \text{ m } (7.70 \text{ ft})$$

$$B = \frac{H}{2} = \frac{3}{2} = 1.5 \text{ m } (4.92 \text{ ft})$$

Water Particle Displacement at *z* = -*d* m

Horizontal WP Displacement, ξ:

$$\xi = A = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta = -\frac{H}{2}\frac{\cosh[2\pi(-d+d)/L]}{\sinh(2\pi d/L)}\sin\theta$$

$$\xi = A = -\frac{H}{2}\frac{1}{\sinh(2\pi d/L)}\sin\theta$$

Vertical WP Displacement, ζ:

$$\zeta = B = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta = \frac{H}{2}\frac{\sinh[2\pi(-d+d)/L]}{\sinh(2\pi d/L)}\cos\theta$$

$$\zeta = B = \frac{H}{2}\frac{\sinh[0]}{\sinh(2\pi d/L)}\cos\theta = 0$$

Water Particle Displacement at $z = -d$ m

When $\boxed{z = - d,}$

$$A = \frac{H}{2\ \sinh(2\pi d/L)} = \frac{3}{2(0.8306)} = 1.81 \text{ m } (5.92 \text{ ft})$$

and, $B = 0$.

Problem 7

GIVEN: A wave in a depth d = 12 meters (39.4 feet), height H = 3 meters (9.8 feet), and a period T = 10 seconds. The corresponding deepwater wave height is H_o = 3.13 meters (10.27 feet).

FIND:

(a) The horizontal and vertical displacement of a water particle from its mean position when z = 0, and when z = -d.

(b) The maximum water particle displacement at an elevation z = 7.5 meters (-24.6 feet) when the wave is in infinitely deep water.

(c) For the deepwater conditions of (b) above, show that the particle displacements are small relative to the wave height when z = $-L_o/2$.

z = -7.5 m

Deep water:
H_o = 3.13 m
T = 10 s

(b) With $H_o = 3.13$ meters and $z = -7.5$ meters (-24.6 feet), evaluate the exponent of e for use in equation (2-24), noting that $L = L_o$,

$$\frac{2\pi z}{L} = \frac{2\pi(-7.5)}{156} = -0.302$$

thus,

$$e^{-0.302} = 0.739$$

Therefore,

$$\boxed{A = B = \frac{H_o}{2} e^{2\pi z/L}} = \frac{3.13}{2}(0.739) = 1.16 \text{ m } (3.79 \text{ ft})$$

The maximum displacement or diameter of the orbit circle would be 2(1.16) = 2.32 meters (7.61 feet).

Problem 7

GIVEN: A wave in a depth d = 12 meters (39.4 feet), height H = 3 meters (9.8 feet), and a period T = 10 seconds. The corresponding deepwater wave height is H_o = 3.13 meters (10.27 feet).

FIND:

(a) The horizontal and vertical displacement of a water particle from its mean position when z = 0, and when z = -d.

(b) The maximum water particle displacement at an elevation z = 7.5 meters (-24.6 feet) when the wave is in infinitely deep water.

(c) For the deepwater conditions of (b) above, show that the particle displacements are small relative to the wave height when z = $-L_o/2$.

Water Particle Displacement at $z = -L_o/2$

(c)

$$z = -\frac{L_o}{2} = \frac{-156}{2} = -\ 78.0\ \mathrm{m}\ (255.9\ \mathrm{ft})$$

$$\frac{2\pi z}{L} = \frac{2\pi(-78)}{156} = -3.142$$

Therefore,

$$e^{-3.142} = 0.043$$

and,

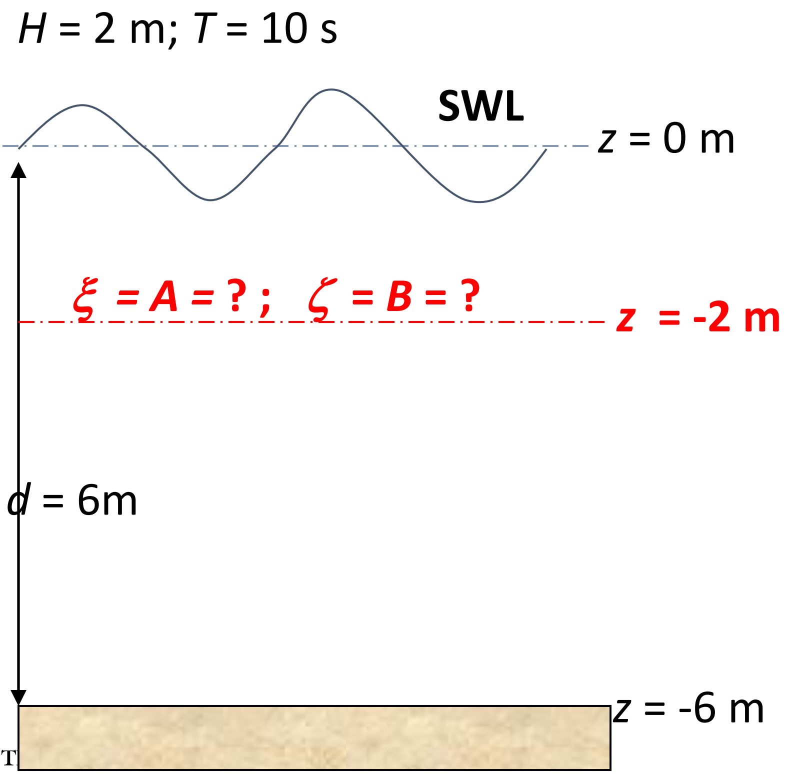
$$A = B = \frac{H_o}{2}\, e^{2\pi z/L} = \frac{3.13}{2}(0.043) = 0.067\ \mathrm{m}\ (0.221\ \mathrm{ft})$$

Thus, the maximum displacement of the particle is 0.067 meters which is small when compared with the deepwater height, $H_O = 3.13$ meters (10.45 feet).

* *

Problem 8

A wave of H = 2 m and T = 10 s propagates in a depth of 6 m. Calculate the horizontal and vertical water particle displacements from its mean position when z = -2 m.



PROCEDURES:

- Find L_o [Ans: 156 m]
- $d/L_o \Rightarrow d/L$ [Ans: 0.0816]
- Identify the water condition
- Determine L [Ans: 73.53 m]
- Find ξ and ζ
  [Ans: A = 1.98m; B= 0.65 m]

Absolute Wave Pressure

Subsurface pressure under a wave is the summation of **dynamic** and **static pressures.**

Total or absolute pressure, *p*’:

$$\underset{\underset{\text{Absolute pressure}}{\downarrow}}{p'} = \underbrace{\rho g \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \frac{H}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)}_{\text{Dynamic pressure (due to acceleration)}} \underbrace{- \rho g z}_{\text{Hydrostatic pressure}} + \underset{\underset{\text{Atmospheric pressure}}{\downarrow}}{P_a}$$

Gage Pressure

Gage pressure, *p*:

$$p = p' - \underset{\substack{\downarrow \\ \text{Atmospheric pressure}}}{p_a} = \underbrace{\rho g \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \frac{H}{2}\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)}_{\text{Dynamic pressure}} \underbrace{- \rho g z}_{\text{Hydrostatic pressure}}$$

The above equation can be simplified as:

$$\boxed{p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z}$$

where

$$\eta = \frac{H}{2}\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

Pressure Response Factor

Gage pressure, *p*:

$$\boxed{p = \rho g \eta \underbrace{\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}}_{\text{Pressure Response Factor} = K_z} - \rho g z}$$

$$\boxed{K_z = \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}}$$

The above equation can be further simplified as:

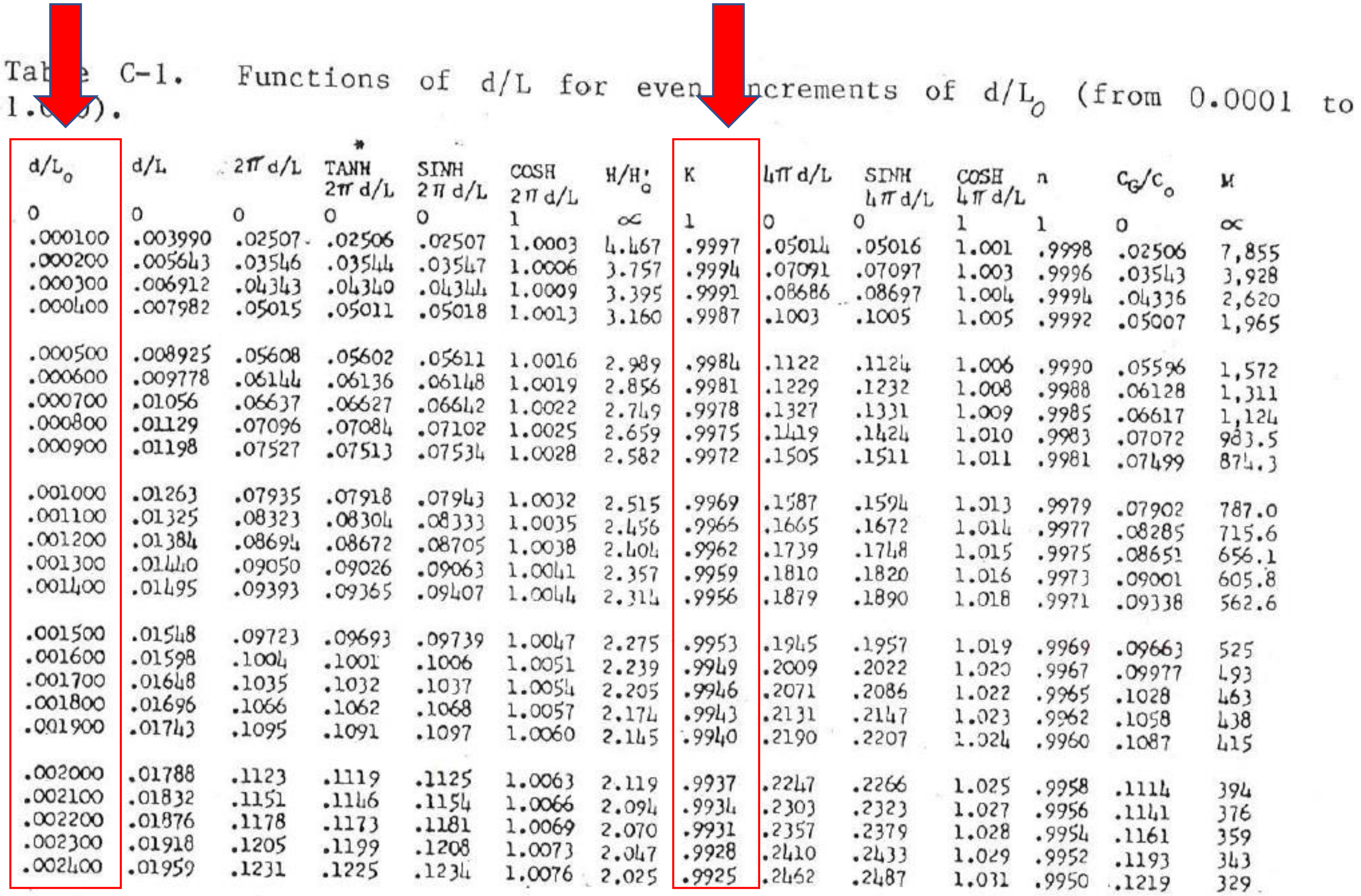
$$\boxed{p = \rho g(\eta K_z - z)}$$

Pressure Response Factor

At sea bottom (z = -d),

$$\boxed{K_z = \frac{\cosh[2\pi(-d+d)/L]}{\cosh(2\pi d/L)} = \frac{1}{\cosh(2\pi d/L)}}$$

Table C-1 (Shore Protection Manual)



Tab e C-1. Functions of d/L for even ncrements of d/L_0 (from 0.0001 to 1.0 0).

d/L_o	d/L	$2\pi d/L$	* TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H'_o	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
0	0	0	0	0	1	∝	1	0	0	1	1	0	∝
.000100	.003990	.02507	.02506	.02507	1.0003	4.467	.9997	.05014	.05016	1.001	.9998	.02506	7,855
.000200	.005643	.03546	.03544	.03547	1.0006	3.757	.9994	.07091	.07097	1.003	.9996	.03543	3,928
.000300	.006912	.04343	.04340	.04344	1.0009	3.395	.9991	.08686	.08697	1.004	.9994	.04336	2,620
.000400	.007982	.05015	.05011	.05018	1.0013	3.160	.9987	.1003	.1005	1.005	.9992	.05007	1,965
.000500	.008925	.05608	.05602	.05611	1.0016	2.989	.9984	.1122	.1124	1.006	.9990	.05596	1,572
.000600	.009778	.06144	.06136	.06148	1.0019	2.856	.9981	.1229	.1232	1.008	.9988	.06128	1,311
.000700	.01056	.06637	.06627	.06642	1.0022	2.749	.9978	.1327	.1331	1.009	.9985	.06617	1,124
.000800	.01129	.07096	.07084	.07102	1.0025	2.659	.9975	.1419	.1424	1.010	.9983	.07072	983.5
.000900	.01198	.07527	.07513	.07534	1.0028	2.582	.9972	.1505	.1511	1.011	.9981	.07499	874.3
.001000	.01263	.07935	.07918	.07943	1.0032	2.515	.9969	.1587	.1594	1.013	.9979	.07902	787.0
.001100	.01325	.08323	.08304	.08333	1.0035	2.456	.9966	.1665	.1672	1.014	.9977	.08285	715.6
.001200	.01384	.08694	.08672	.08705	1.0038	2.404	.9962	.1739	.1748	1.015	.9975	.08651	656.1
.001300	.01440	.09050	.09026	.09063	1.0041	2.357	.9959	.1810	.1820	1.016	.9973	.09001	605.8
.001400	.01495	.09393	.09365	.09407	1.0044	2.314	.9956	.1879	.1890	1.018	.9971	.09338	562.6
.001500	.01548	.09723	.09693	.09739	1.0047	2.275	.9953	.1945	.1957	1.019	.9969	.09663	525
.001600	.01598	.1004	.1001	.1006	1.0051	2.239	.9949	.2009	.2022	1.020	.9967	.09977	493
.001700	.01648	.1035	.1032	.1037	1.0054	2.205	.9946	.2071	.2086	1.022	.9965	.1028	463
.001800	.01696	.1066	.1062	.1068	1.0057	2.174	.9943	.2131	.2147	1.023	.9962	.1058	438
.001900	.01743	.1095	.1091	.1097	1.0060	2.145	.9940	.2190	.2207	1.024	.9960	.1087	415
.002000	.01788	.1123	.1119	.1125	1.0063	2.119	.9937	.2247	.2266	1.025	.9958	.1114	394
.002100	.01832	.1151	.1146	.1154	1.0066	2.094	.9934	.2303	.2323	1.027	.9956	.1141	376
.002200	.01876	.1178	.1173	.1181	1.0069	2.070	.9931	.2357	.2379	1.028	.9954	.1161	359
.002300	.01918	.1205	.1199	.1208	1.0073	2.047	.9928	.2410	.2433	1.029	.9952	.1193	343
.002400	.01959	.1231	.1225	.1234	1.0076	2.025	.9925	.2462	.2487	1.031	.9950	.1219	329

Subsurface Pressure Under a Wave

Shallow water:

$$p = \rho g(\eta - z)$$

Transitional water:

$$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$$

Deep water:

$$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$$

Application of Pressure Measurement

To determine the height of surface waves based on subsurface measurements of pressure, it is convenient to rewrite the equation

$$p = \rho g (\eta K_z - z)$$

$$\boxed{\eta = \frac{N(p + \rho g z)}{\rho g K_z}}$$

z = The depth below the SWL of the pressure gage
N = Correction factor equal to unity if the linear theory applied

$$N = f\,(T, d, a)$$

Problem 8

GIVEN: An average maximum pressure p = 124 kilonewtons per square meter is measured by a subsurface pressure gage located in salt water 0.6 meter (1.97 feet) above the bed in water depth d = 12 meters (39 feet). The average frequency f = 0.0666 cycles per second (hertz).

FIND: The height of the wave H assuming that linear theory applies and the average frequency corresponds to the average wave amplitude.

SOLUTION:

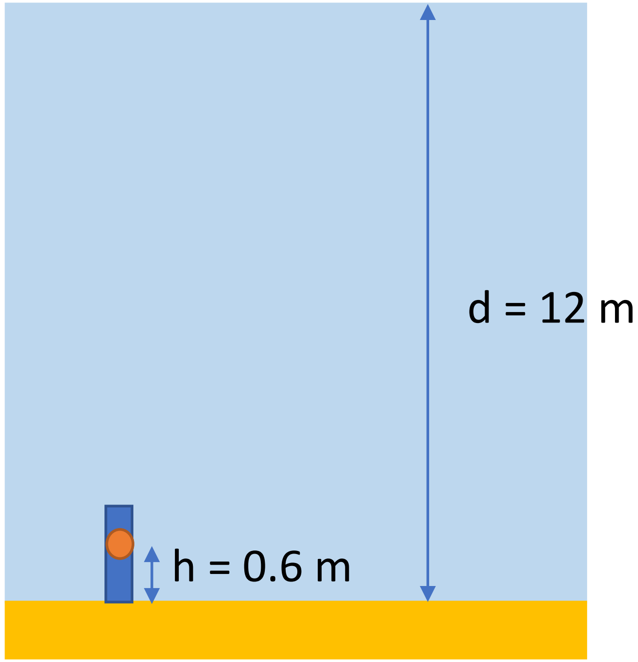
$$T = \frac{1}{f} = \frac{1}{(0.0666)} \approx 15 \text{ s}$$

$$L_o = 1.56T^2 = 1.56(15)^2 = 351 \text{ m} \ (1152 \text{ ft})$$

$$\frac{d}{L_o} = \frac{12}{351} \approx 0.0342$$

From Table C-1 of Appendix C, entering with d/L_o,

$$\frac{d}{L} = 0.07651$$



$$\eta = \frac{N(p+\rho g z)}{\rho g K_z}$$

Density of seawater = 1.025 kg/m^3

hence,

$$L = \frac{12}{(0.07651)} = 156.8 \text{ m } (515 \text{ ft})$$

and

$$\cosh\left(\frac{2\pi d}{L}\right) = 1.1178$$

Therefore, from equation (2-29)

$$K_z = \frac{\cosh[2\pi(z + d)/L]}{\cosh(2\pi d/L)} = \frac{\cosh[2\pi[-11.4 + 12)/156.8]}{1.1178} = \frac{1.0003}{1.1178} = 0.8949$$

Since $\eta = a = H/2$ when the pressure is maximum (under the wave crest), and $N = 1.0$ since linear theory is assumed valid,

$$\frac{H}{2} = \frac{N(p + \rho g z)}{\rho g K_z} = \frac{1.0\ [124 + (10.06)\ (-11.4)]}{(10.06)\ (0.8949)} = 1.04 \text{ m } (3.44 \text{ ft})$$

Therefore,

$$H = 2(1.04) = 2.08 \text{ m } (6.3 \text{ ft})$$

Note that the tabulated value of K in Appendix C, Table C-1, could not be used since the pressure was not measured at the bottom.

Wave Energy

E $_{total}$ = E $_{potential}$ + E $_{kinetic}$

Total energy of a wave system (J/m^2)

$$E = \frac{1}{8}\rho g H_i^{\,2} b$$

Due to fluid mass of the wave crest

$$E_p = \frac{1}{16}\rho g H_p^{\,2} b$$

Due to water particles velocities associated with wave motion

$$E_k = \frac{1}{16}\rho g H_k^{\,2} b$$

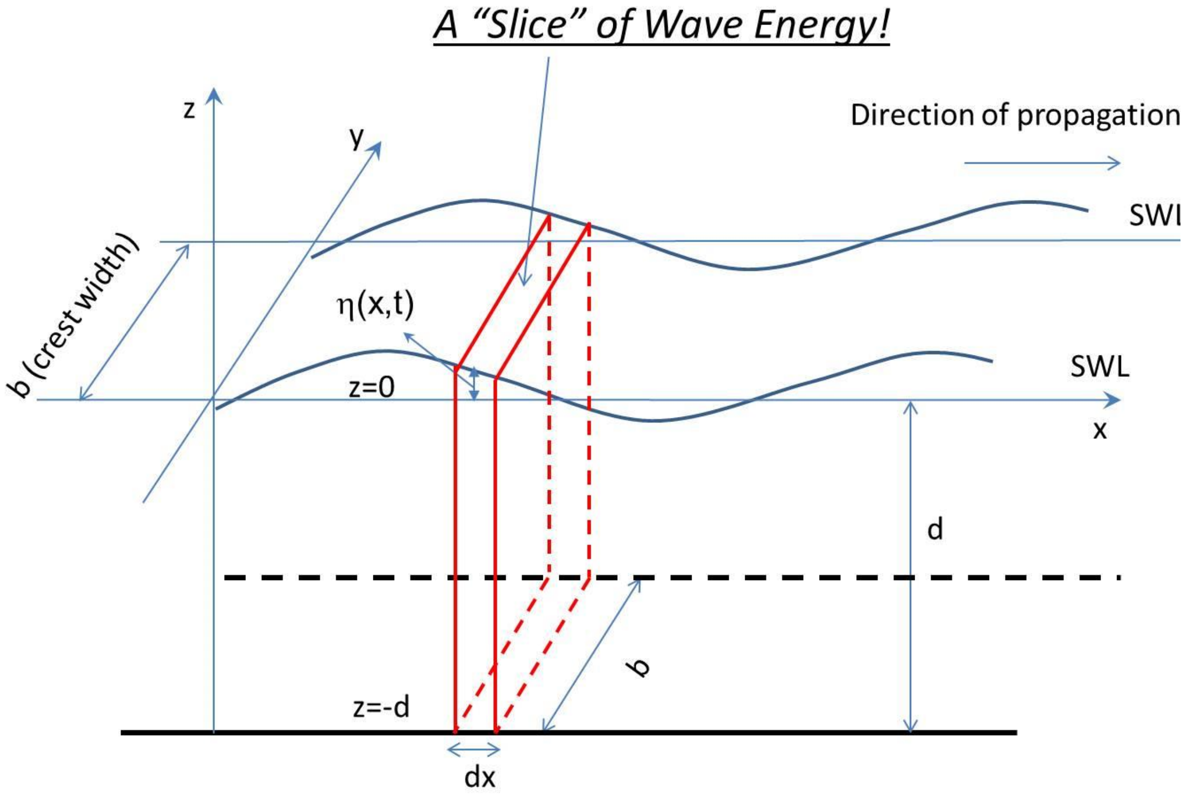
Total average wave energy per unit surface area, termed the specific energy or energy density, is given by

$$\boxed{\overline{E} = \frac{E}{b} = \frac{\rho g H^2}{8}}$$

Unit: J/m^2 per m of crest width

Wave Energy Flux

The rate at which energy is transmitted in the direction of wave propagation across a vertical plane perpendicular to the direction of wave advance and extending down the entire depth.



Wave Energy Flux

Assuming linear wave theory holds, the average energy flux per unit wave crest width transmitted across a vertical plane perpendicular to the direction of wave advance is

$$\boxed{\overline{P} = \overline{E}C_g = \overline{E}(n.C)}$$

$\overline{P}$ = **Average energy flux (or wave power) per unit wave crest width**

(*Nm/s* per *m* of crest width or *W* per *m* of crest width)

E = **Total wave energy (J/m^2 per m of crest width)**

C_g = $n.\,C$ = **Group velocity**

Wave Energy Flux in Inclined Angle

If a vertical plane is taken other than perpendicular to the direction of wave advance,

$$\boxed{\overline{P} = \overline{E} C_g \sin\theta}$$

where θ is the angle between the plan across which the energy is being transmitted and the direction of wave advance.

Wave Energy Flux in Different Water Depths

$$\boxed{\overline{P} = \overline{E}C_G = \overline{E}(n.C)}$$

Group velocity factor, *n*:

$$\boxed{n = \frac{1}{2}\left[1 + \frac{4\pi d / L}{\sinh(4\pi d / L)}\right]}$$

Deep water,	$n = 0.5$;	$C_G = 0.5C$
Shallow water,	$n = 1$;	$C_G = C$
Transitional water,	$0.5 < n < 1$	

For deep water, $n = ½$ $\Rightarrow$ $\overline{P} = \frac{1}{2}\overline{E_o C_o}$

For shallow water, $n = 1$ $\Rightarrow$ $\overline{P} = \overline{E}C_g = \overline{E}C$

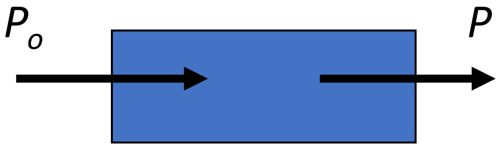
For transitional water, $½ < n < 1$ $\Rightarrow$ $\overline{P} = \overline{E}C_g = \overline{E}(n.C)$

Wave Energy Flux

Conservation of Energy

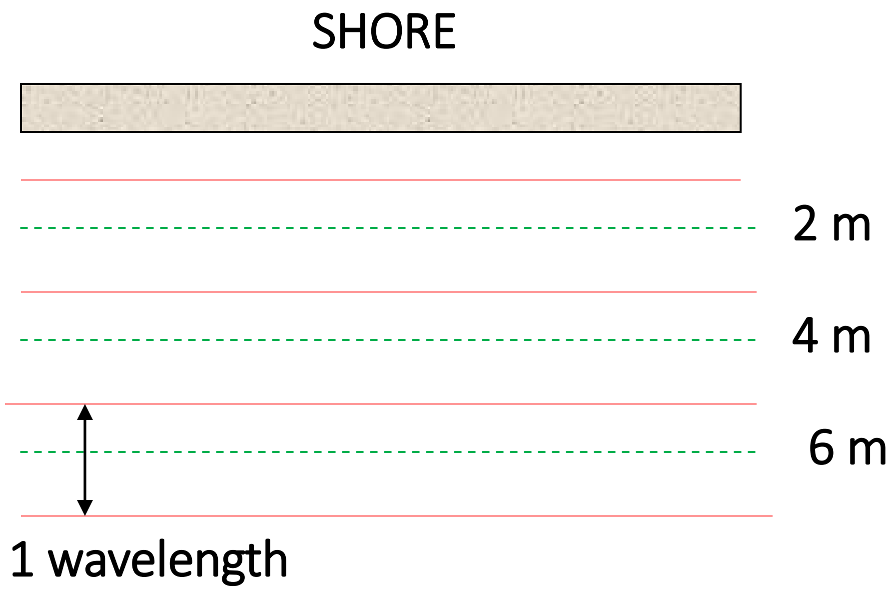
Amount of energy entering a region which waves are passing will equal amount leaving the region provided no energy is added or removed from the system. Therefore, when the waves are moving so that their crests are parallel to the bottom contours,

$$P_o = P$$

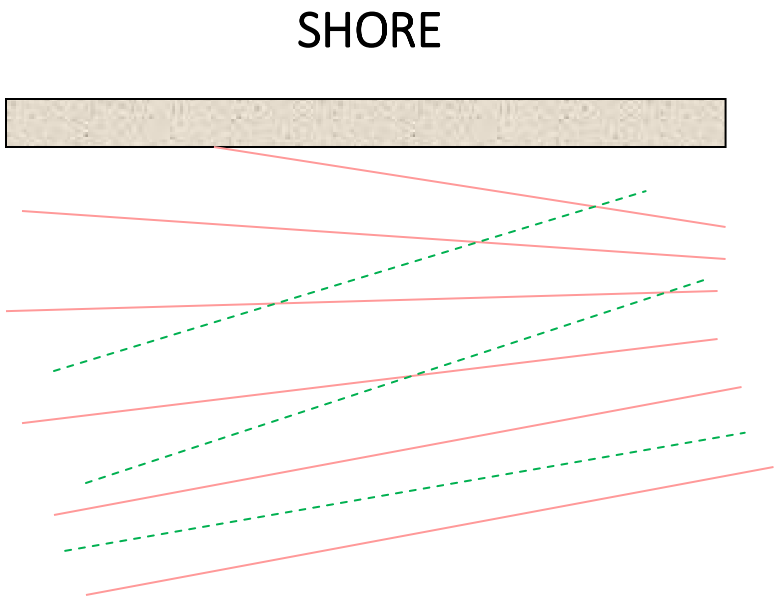


$$\frac{1}{2}\overline{E_o}C_o = \overline{E}nC$$

Wave Refraction



No refraction



With refraction

Wave Energy Flux

Deepwater

Transitional/
shallow water

$P_o = P$

$$\frac{1}{2}\overline{E_o C_o} = \overline{E}nC$$

$$\overline{E}_o = \frac{\rho g H_o^{\ 2}}{8}$$

$$C_o = \frac{gT}{2\pi}$$

$$C = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$$

$$\overline{E} = \frac{\rho g H^{\ 2}}{8}$$

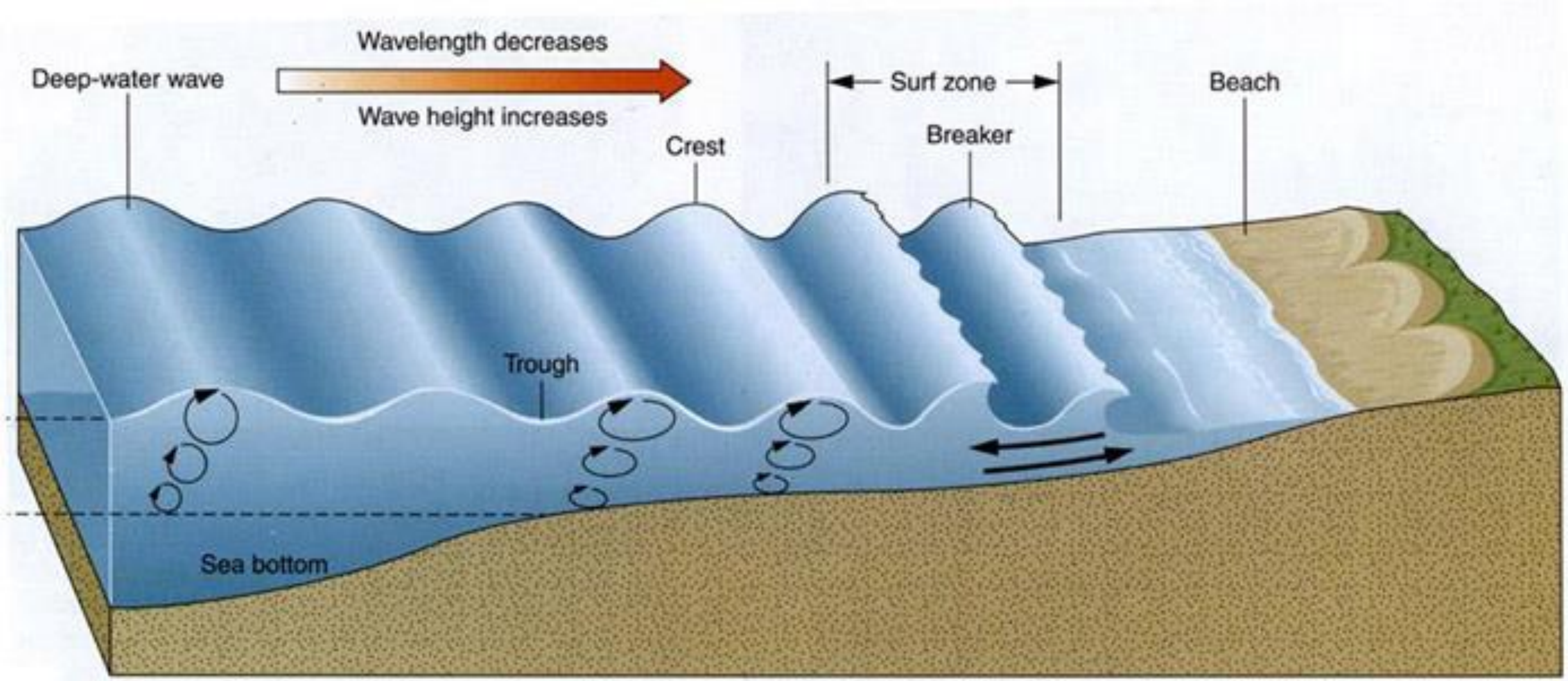
$$\frac{1}{2}.\frac{\rho g H_o^2}{8}.C_o = \frac{\rho g H^2}{8}nC$$

$$\left(\frac{H}{H_o}\right)^2 = \frac{1}{2}.\frac{1}{n}.\frac{C_o}{C}$$

K_s = Shoaling coeff.

The ratio of the height of a wave in water of any depth to its height in deepwater with the effects of refraction, friction and percolation eliminated.

Wave Shoaling



(Plummer et al., 2001)

Problem 9

GIVEN: A deepwater oscillatory wave with a wavelength L_O = 156 meters (512 feet), a height H_O = 2 meters (6.56 feet), and a celerity C_O = 15.6 meters per second, moving shoreward with its crest parallel to the depth contours. Any effects due to reflection from the beach are negligible.

FIND:

(a) Derive a relationship between the wave height in any depth of water and the wave height in deep water, assuming that wave energy flux per unit crest width is conserved as a wave moves from deep water into shoaling water.

(b) Calculate the wave height for the given wave when the depth is 3 meters (9.84 feet).

(c) Determine the rate at which energy per unit crest width is transported toward the shoreline and the total energy per unit width delivered to the shore in 1 hour by the given waves.

SOLUTION:

(a) Since the wave crests are parallel to the bottom contours, refraction does not occur, therefore $H_o = H'_o$ (see Sec. III).

From equation (2-43),

$$\frac{1}{2} \bar{E}_o C_o = n\bar{E}C$$

The expressions for $\bar{E}_o$ and $\bar{E}$ are

$$\bar{E}_o = \frac{\rho g H_o'^2}{8}$$

and

$$\bar{E} = \frac{\rho g H^2}{8}$$

where H'_o represents the wave height in deep water if the wave is not refracted. Substituting into the above equation gives

$$\frac{1}{2} C_o \frac{\rho g H_o'^2}{8} = nC \frac{\rho g H^2}{8}$$

Therefore,

$$\left(\frac{H}{H'_o}\right)^2 = \frac{1}{2}\,\frac{1}{n}\,\frac{C_o}{C}$$

and since from equations (2-3) and (2-6)

$$\frac{C}{C_o} = \tanh\left(\frac{2\pi d}{L}\right)$$

and from equation (2-35) where

$$n = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right]$$

$$\frac{H}{H'_o} = \sqrt{\frac{1}{\tanh(2\pi d/L)}\;\frac{1}{\left[1 + \dfrac{(4\pi d/L)}{\sinh(4\pi d/L)}\right]}} = K_s \tag{2-44}$$

where K_s or H/H'_o is termed the *shoaling coefficient.* Values of H/H'_o as a function of d/L_o and d/L have been tabulated in Tables C-1 and C-2 of Appendix C.

Problem 9

GIVEN: A deepwater oscillatory wave with a wavelength L_o = 156 meters (512 feet), a height H_o = 2 meters (6.56 feet), and a celerity C_o = 15.6 meters per second, moving shoreward with its crest parallel to the depth contours. Any effects due to reflection from the beach are negligible.

FIND:

(a) Derive a relationship between the wave height in any depth of water and the wave height in deep water, assuming that wave energy flux per unit crest width is conserved as a wave moves from deep water into shoaling water.

(b) Calculate the wave height for the given wave when the depth is 3 meters (9.84 feet).

(c) Determine the rate at which energy per unit crest width is transported toward the shoreline and the total energy per unit width delivered to the shore in 1 hour by the given waves.

(b) For the given wave, $d/L_o = 3/156 = 0.01923$. From Table C-1 or from an evaluation of equation (2-44) above,

$$\frac{H}{H'_o} = 1.237$$

Therefore,

$$H = 1.237(2) = 2.474 \text{ m } (8.117 \text{ ft})$$

Problem 9

GIVEN: A deepwater oscillatory wave with a wavelength L_o = 156 meters (512 feet), a height H_o = 2 meters (6.56 feet), and a celerity C_o = 15.6 meters per second, moving shoreward with its crest parallel to the depth contours. Any effects due to reflection from the beach are negligible.

FIND:

(a) Derive a relationship between the wave height in any depth of water and the wave height in deep water, assuming that wave energy flux per unit crest width is conserved as a wave moves from deep water into shoaling water.

(b) Calculate the wave height for the given wave when the depth is 3 meters (9.84 feet).

(c) Determine the rate at which energy per unit crest width is transported toward the shoreline and the total energy per unit width delivered to the shore in 1 hour by the given waves.

(c) The rate at which energy is being transported toward shore is the wave energy flux.

$$\overline{P} = \frac{1}{2}\overline{E}_o C_o = n\overline{E}C$$

Since it is easier to evaluate the energy flux in deep water, the left side of the above equation will be used.

$$\overline{P} = \frac{1}{2}\overline{E}_o C_o = \frac{1}{2}\,\frac{\rho g (H'_o)^2\ 15.6}{8} = \frac{1}{2}\,\frac{10{,}050(2)^2}{8}\,15.6$$

$\overline{P}$ = 39,195 N•m/s per m of wave crest

This represents an expenditure of

$$39{,}195\ \frac{N\cdot m}{s} \times 3600\ \frac{s}{h} = 14.11 \times 10^7\ J$$

of energy each hour on each meter of beach (31.72 x 10^6 foot-pounds each hour on each foot of beach).

* *

Problem 10

A deepwater wave with a height $H_o = 2$ m and period $T = 10$ s, moving shoreward with its crest parallel to the depth contours (with no refraction). Determine

a. the wave height for the given wave when $d = 7$ m

b. the rate at which energy per unit crest width is transported toward the shoreline.

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LIVE FORUM

IMPACTS OF CLIMATE CHANGE ON COASTAL SUSTAINABILITY

SEPTEMBER 15, 2021 | 2:00 P.M. | MICROSOFT TEAMS

YBhg. Dato' Mohamed Zin Bin Yusop
Director of Perak Forestry Department

Dato' Paduka Ir. (Dr.) Hj. Keizrul Bin Abdullah
Director of Wetlands International Malaysia

Tuan Ir. Baharuddin Bin Abdullah
Director of Department of Irrigation & Drainage (JPS) Perak

Ir. Dr. Lee Hin Lee
Director of Coastal Management & Oceanography Research Centre (NAHRIM)

Dr. Ahmad Aldrie Amir
Senior Lecturer/ Research Fellow at Institute for Environment & Development (LESTARI) of Universiti Kebangsaan Malaysia (UKM)

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The details of the forum are as follows:

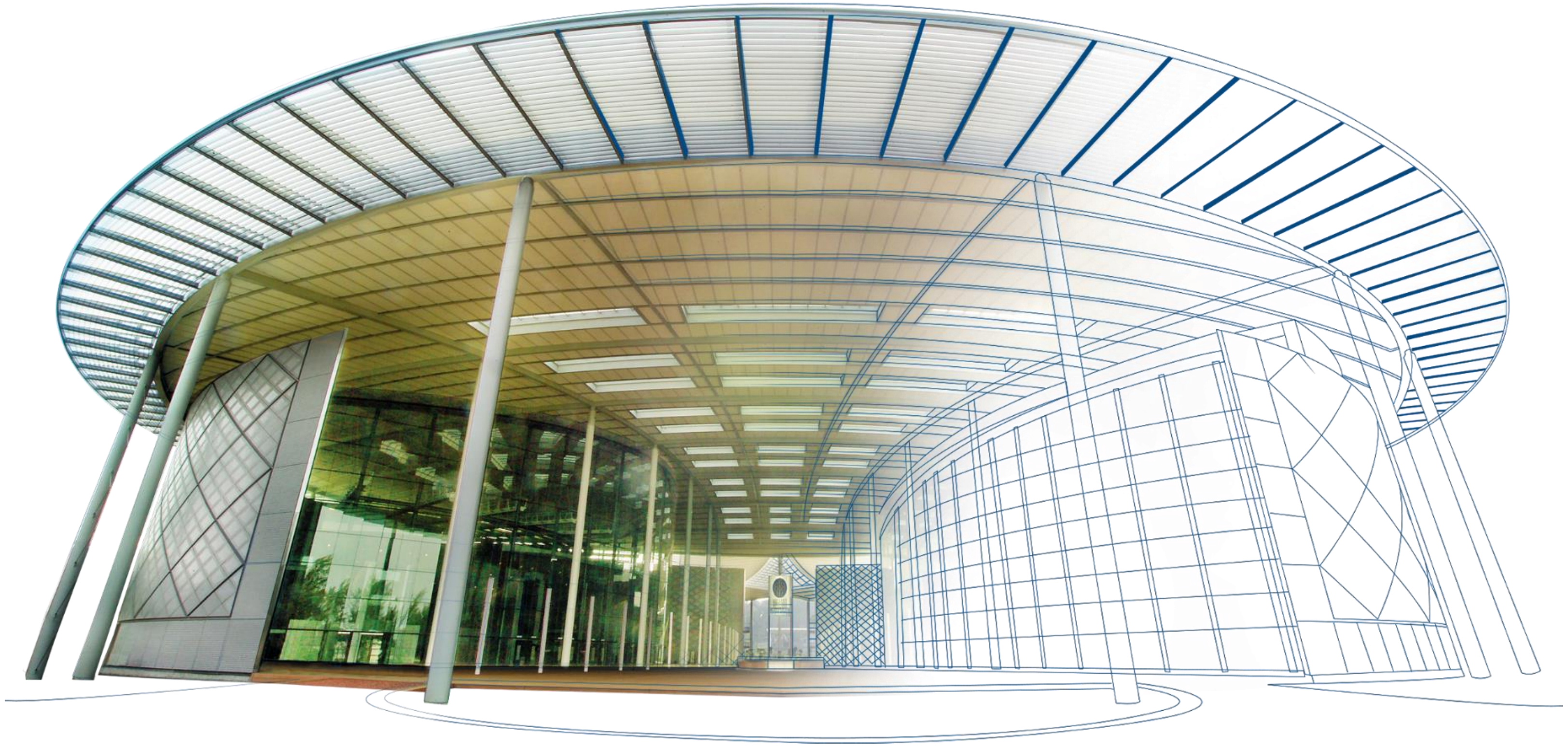
📅Date: 15 September 2021 (Wednesday)
⏰Time: 2.00 pm
💻Platform: Microsoft Teams
📌Forum topic: Impacts of Climate Change on Coastal Sustainability

🔗 Registration link: https://forms.office.com/r/RdMawdeMH7

📌The forum is completely free of charge.

📌All registered participants will receive e-certificates.

📌Stand a chance to win e-vouchers by answering a quiz at end of the forum session.



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