



TOPIC 2

WAVES



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- Part 1: Introduction to Ocean Waves
- Part 2: Linear Wave Theory
- Part 3: Nearshore Wave Transformation

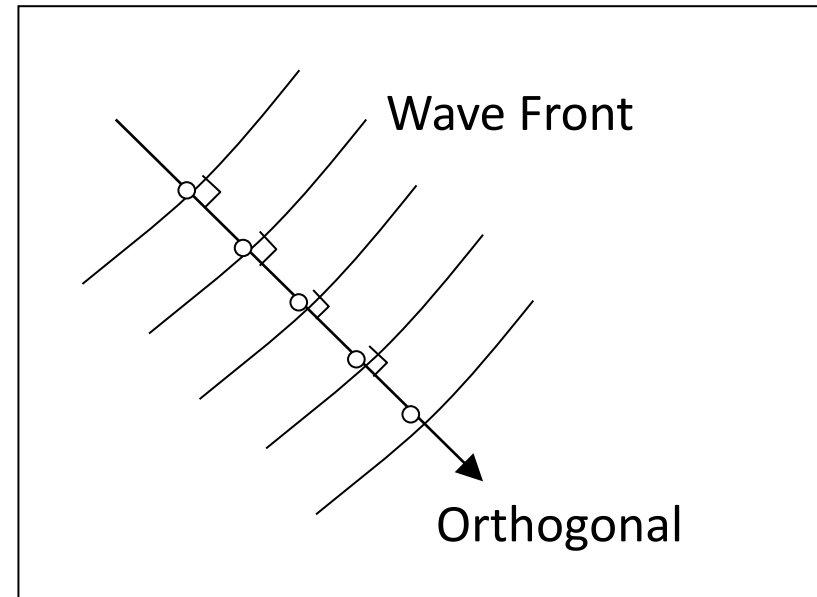
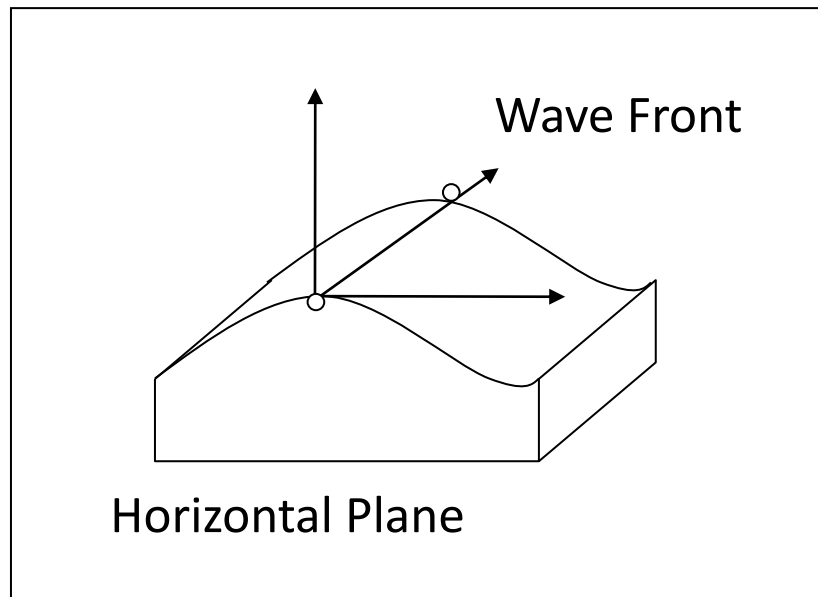
Upon completion of this topic, students should be able:

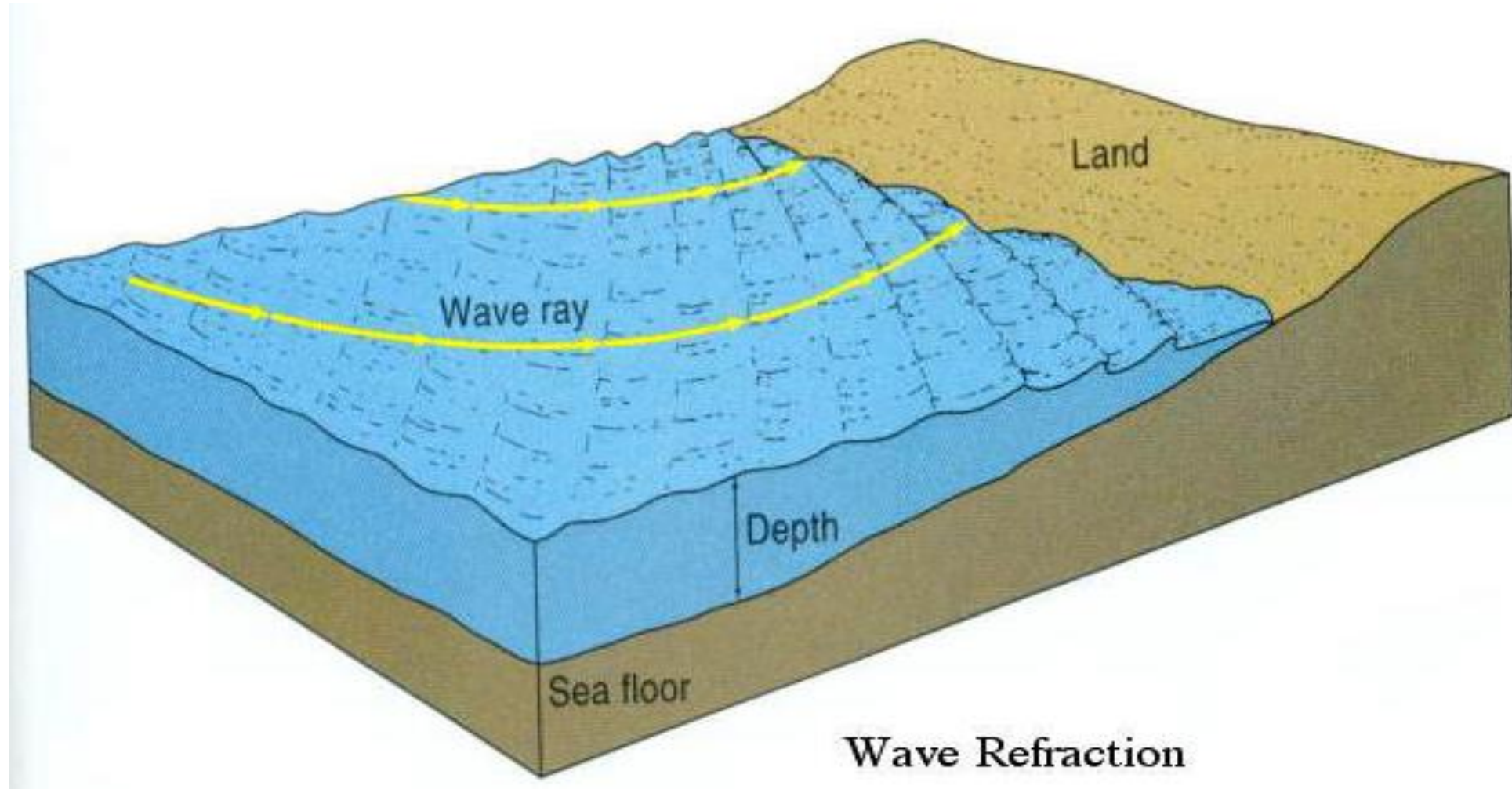
- To assess wave refraction effect at near-shore
- To perform wave refraction analysis

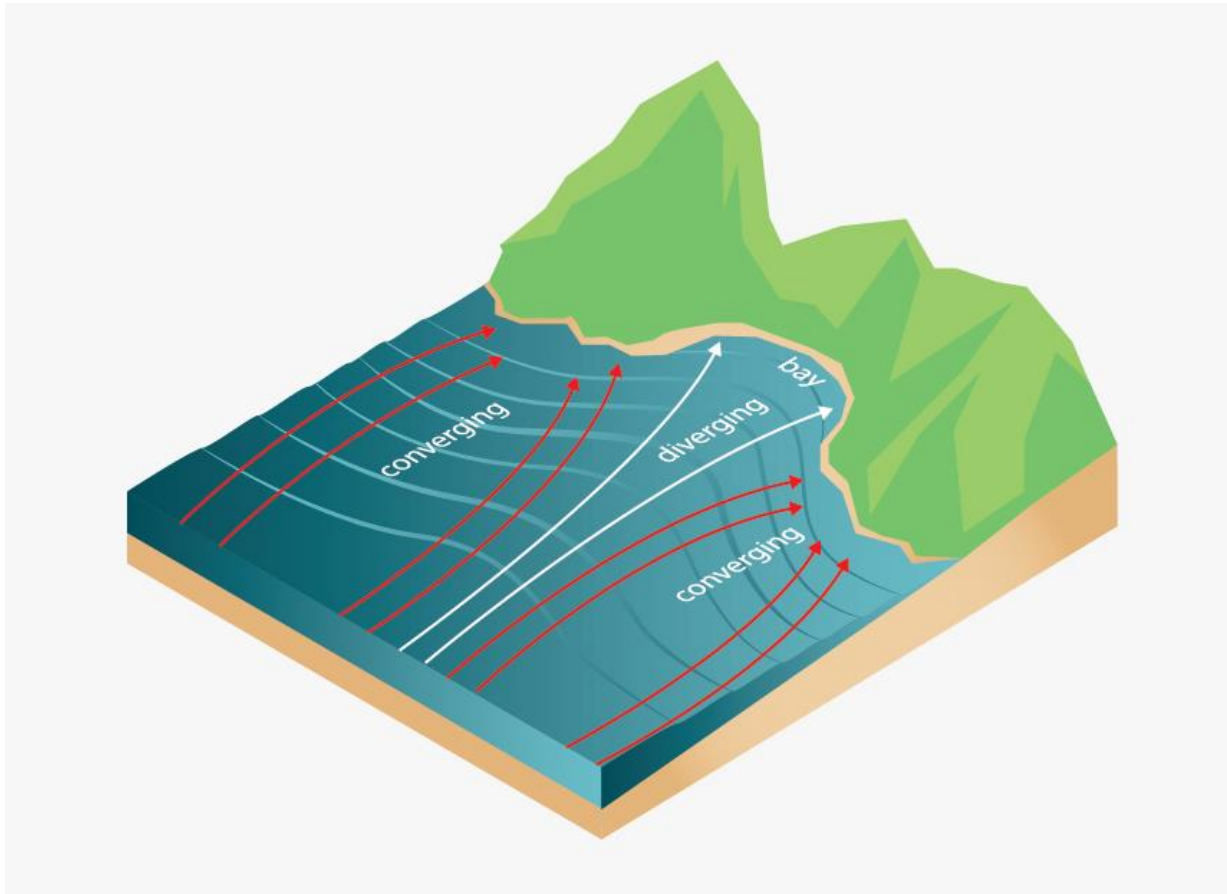


Wave front - A curve in the horizontal plane through adjacent crest points.

Wave orthogonal/ray – Path perpendicular to the wave fronts at every point.



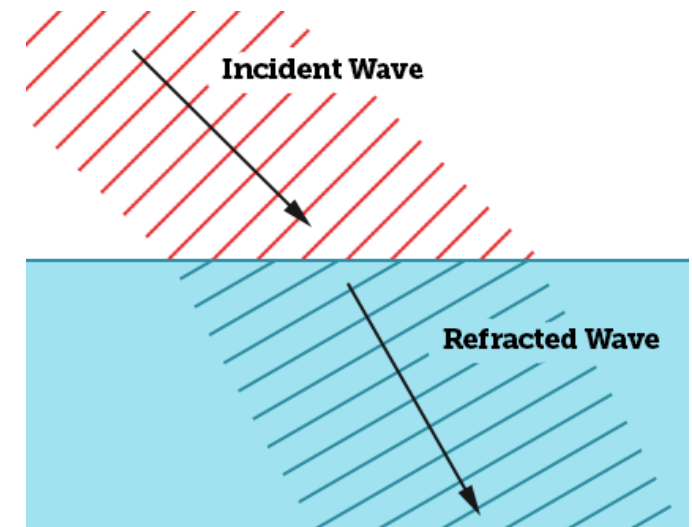




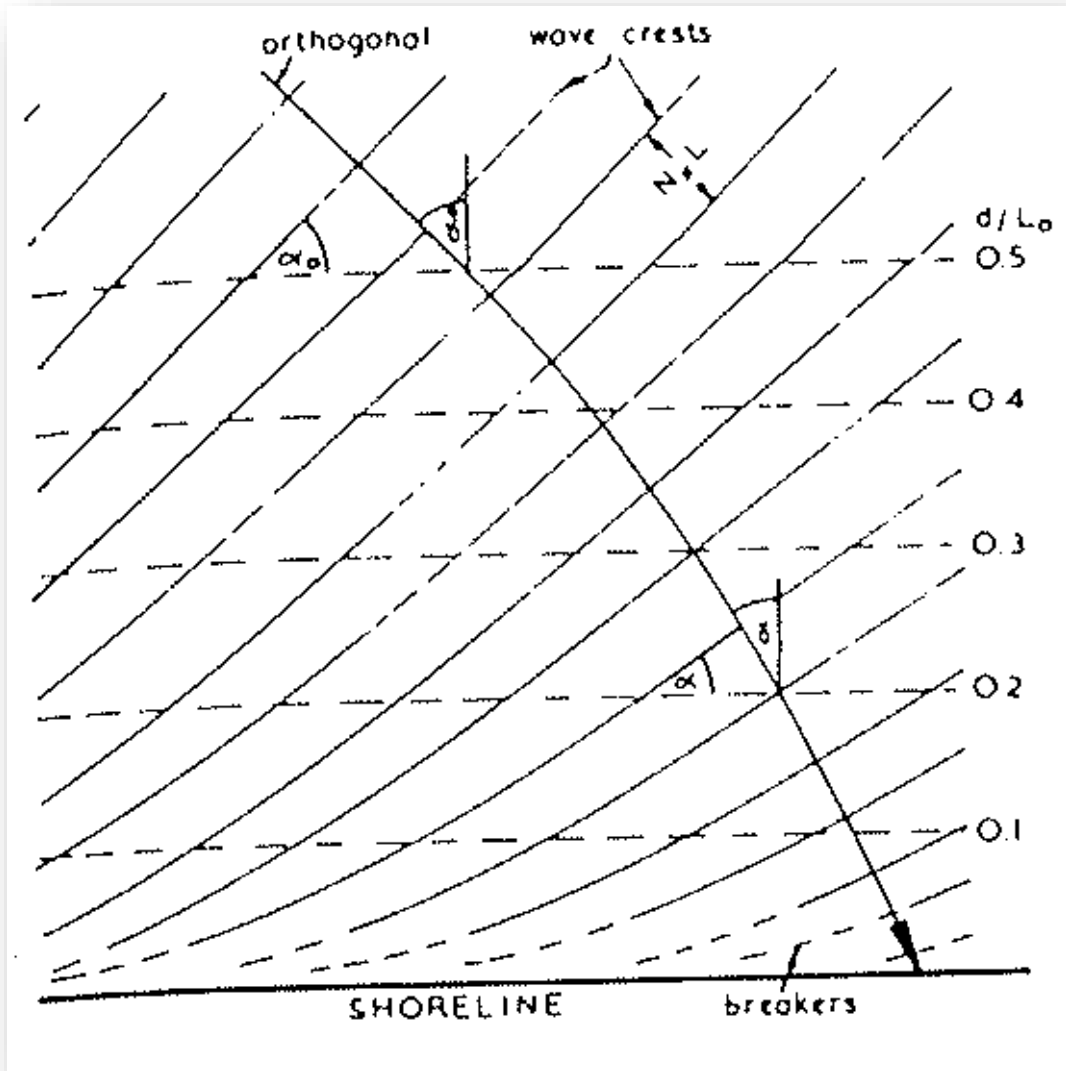
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Wave refraction is the **bending effect of wave crest (wave front)** in order to align with **bottom contours** as waves are moving over **different depths**.

The wave ray becomes more **perpendicular** to the shore.



OBLIQUE WAVES REFRACTING ACROSS A UNIFORMLY SLOPED SHELF



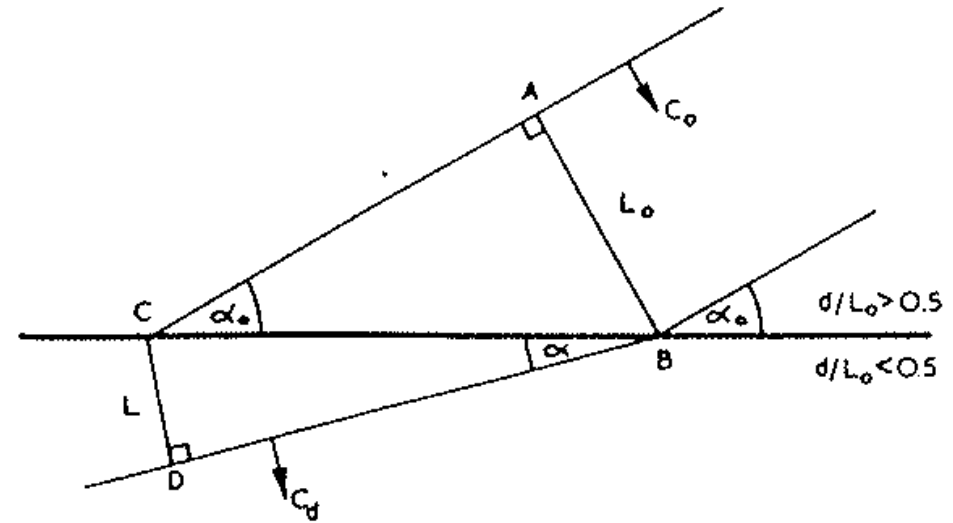
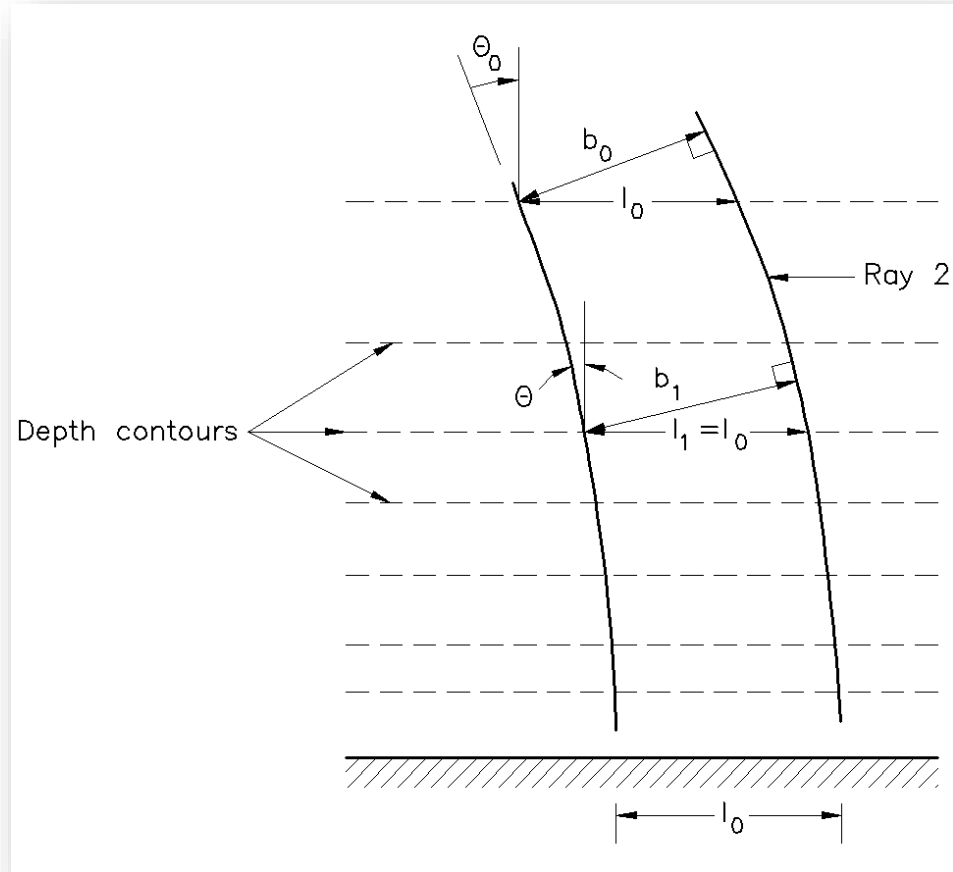
- α_0 is the angle between wave crest and bed contour **OR** between the orthogonal and a normal to the bed contour.

At deep water ($d/L > 0.5$):

- The wave celerity (C_0) is constant.

At intermediate depths ($d/L < 0.5$):

- The wave crests bend.
- α_0 reduce to α .



Snell's Law:

$$\frac{C}{C_o} = \tanh \frac{2\pi d}{L} = \frac{\sin \alpha}{\sin \alpha_o} = \frac{L}{L_o}$$

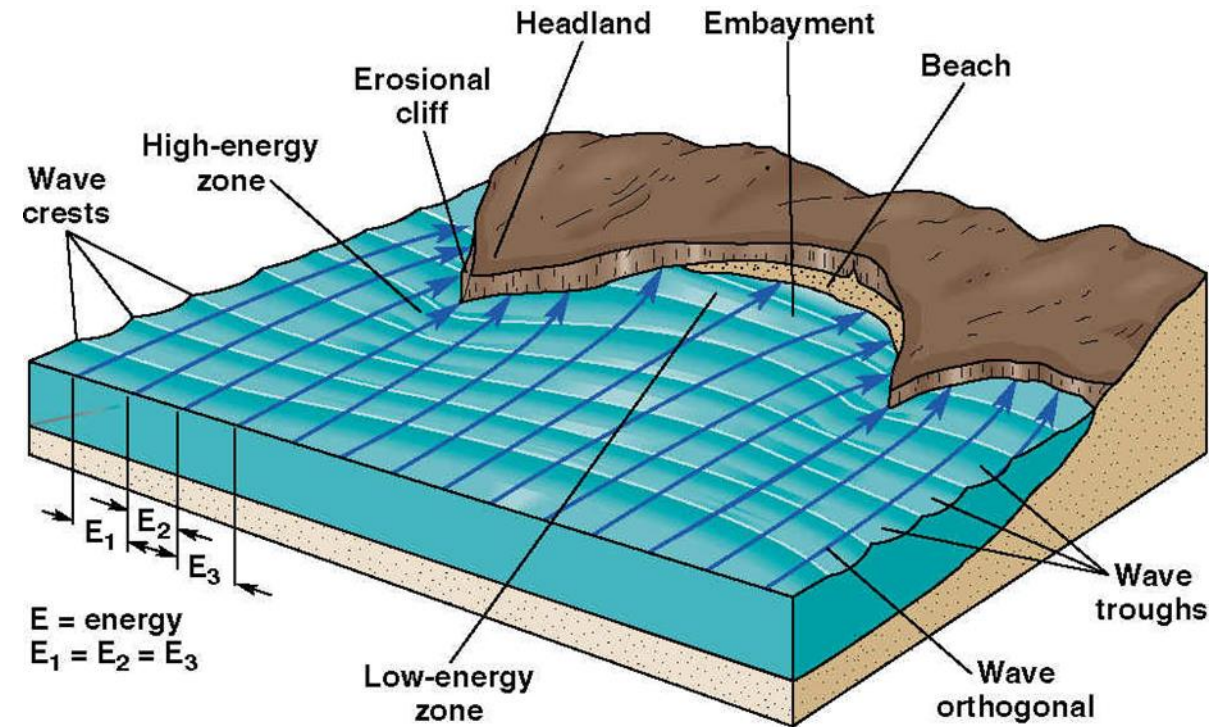
Table C-1 (Shore Protection Manual, 1984)



d/L_o	d/L	$2\pi d/L$	$\frac{\sin \theta}{\sin \theta_o} = \frac{C}{C_o} = \frac{L}{L_o} = \tanh \frac{2\pi d}{L}$	SINH $\frac{2\pi d}{L}$	COSH $\frac{2\pi d}{L}$	H/H_o	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_g/C_o	M
.3300	.3394	2.133	.9723	4.159	4.277	.9583	.2338	4.265	35.58	35.59	.5599	.5444	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.9586	.2325	4.277	35.99	36.00	.5594	.5441	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.9589	.2312	4.288	36.42	36.43	.5589	.5438	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.9592	.2299	4.300	36.84	36.85	.5584	.5434	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.9595	.2286	4.311	37.25	37.27	.5578	.5431	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.9598	.2273	4.323	37.70	37.72	.5573	.5427	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.9601	.2260	4.335	38.14	38.15	.5568	.5424	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.9604	.2247	4.346	38.59	38.60	.5563	.5421	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.9607	.2235	4.358	39.02	39.04	.5558	.5417	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.9610	.2222	4.369	39.48	39.49	.5553	.5414	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.9613	.2210	4.381	39.95	39.96	.5548	.5411	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.9615	.2198	4.392	40.40	40.41	.5544	.5408	5.185
.3420	.3504	2.202	.9758	4.466	4.576	.9618	.2185	4.404	40.87	40.89	.5539	.5405	5.182
.3430	.3514	2.208	.9761	4.492	4.602	.9621	.2173	4.416	41.36	41.37	.5534	.5402	5.179
.3440	.3523	2.214	.9764	4.521	4.630	.9623	.2160	4.427	41.85	41.84	.5529	.5399	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.9626	.2148	4.439	42.33	42.34	.5524	.5396	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.9629	.2136	4.451	42.83	42.84	.5519	.5392	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.9632	.2124	4.462	43.34	43.35	.5515	.5389	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.9635	.2111	4.474	43.85	43.86	.5510	.5386	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.9638	.2099	4.486	44.37	44.40	.5505	.5383	5.162
.3500	.3579	2.249	.9780	4.685	4.791	.9640	.2087	4.498	44.89	44.80	.5501	.5380	5.159
.3510	.3588	2.255	.9782	4.713	4.818	.9643	.2076	4.509	45.42	45.43	.5496	.5377	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.9646	.2064	4.521	45.95	45.96	.5492	.5374	5.154

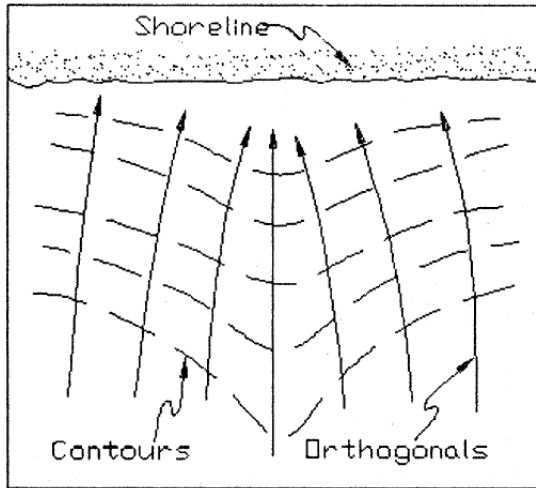
The amount of reduction or amplification of waves due to refraction depends on:

- bathymetry
- the initial angle of approach
- wave period

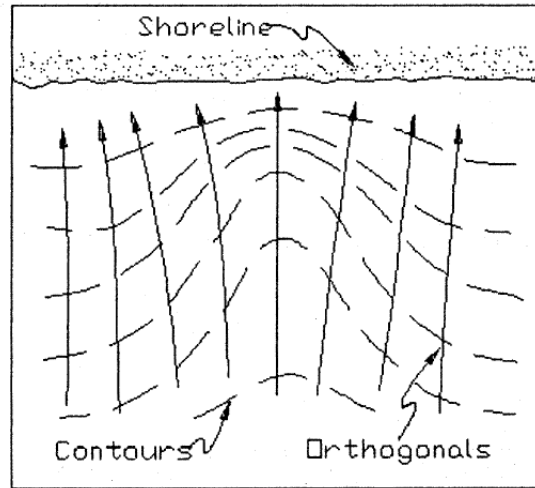


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WAVE REFRACTION



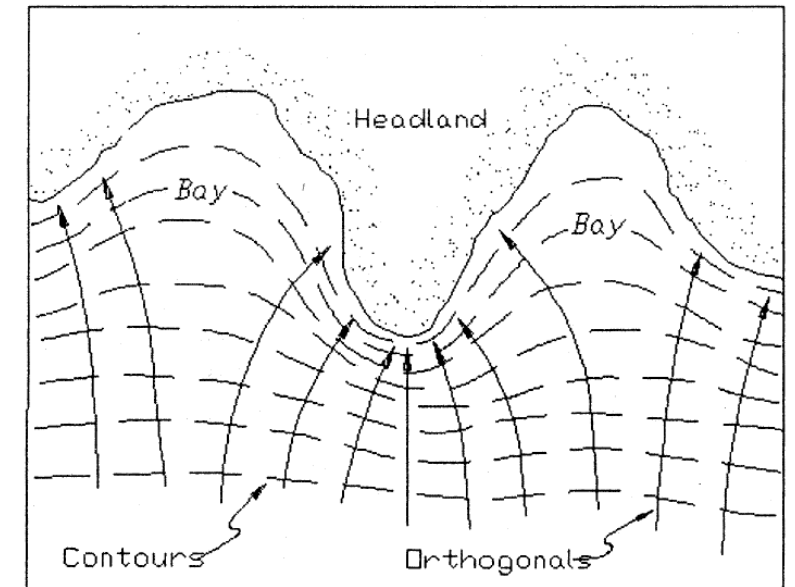
(a)



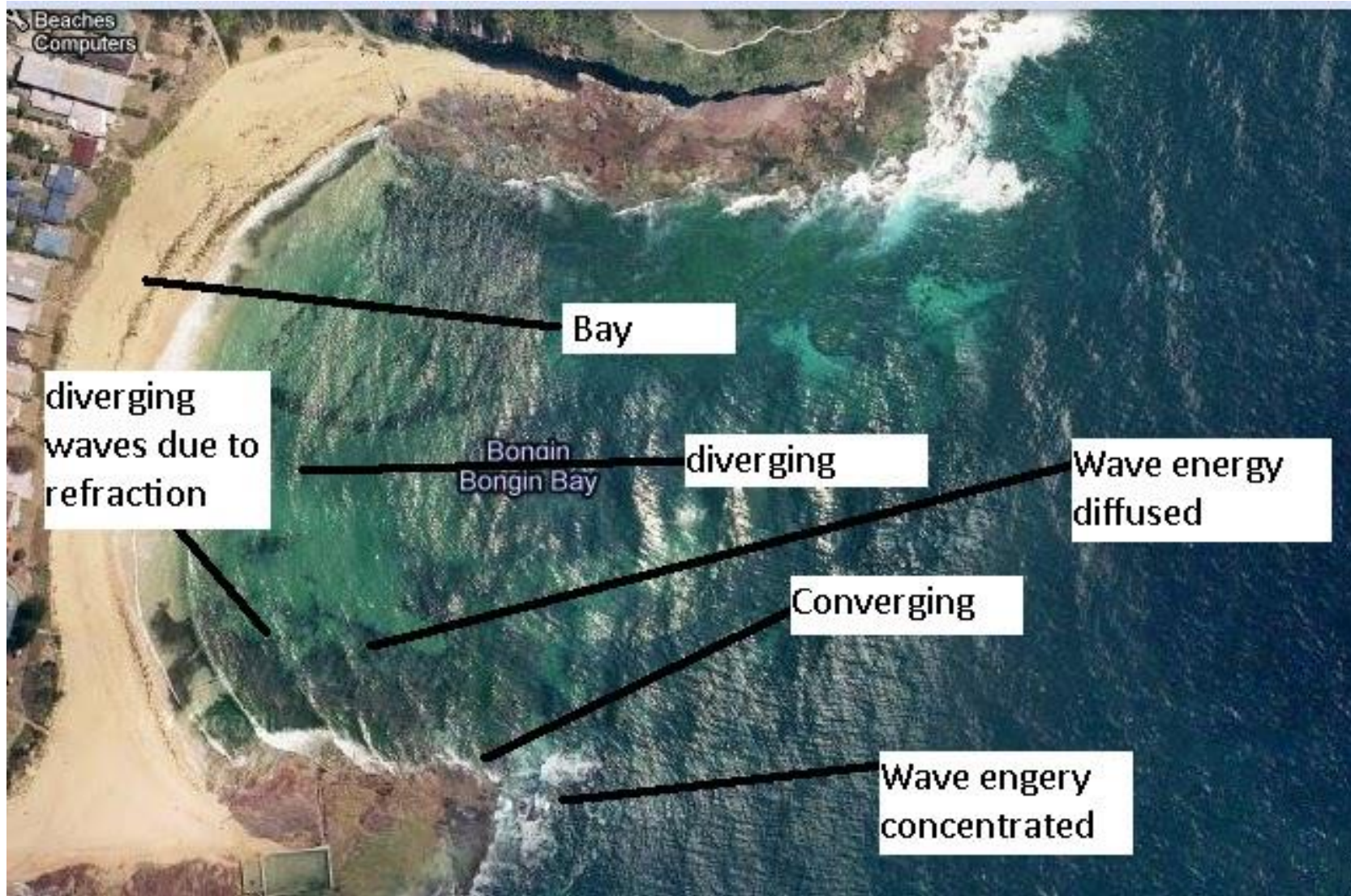
(b)

Wave convergence or divergence, which is determined by the **shape of the bottom topography**, causes energy to be **concentrated or spread out**.

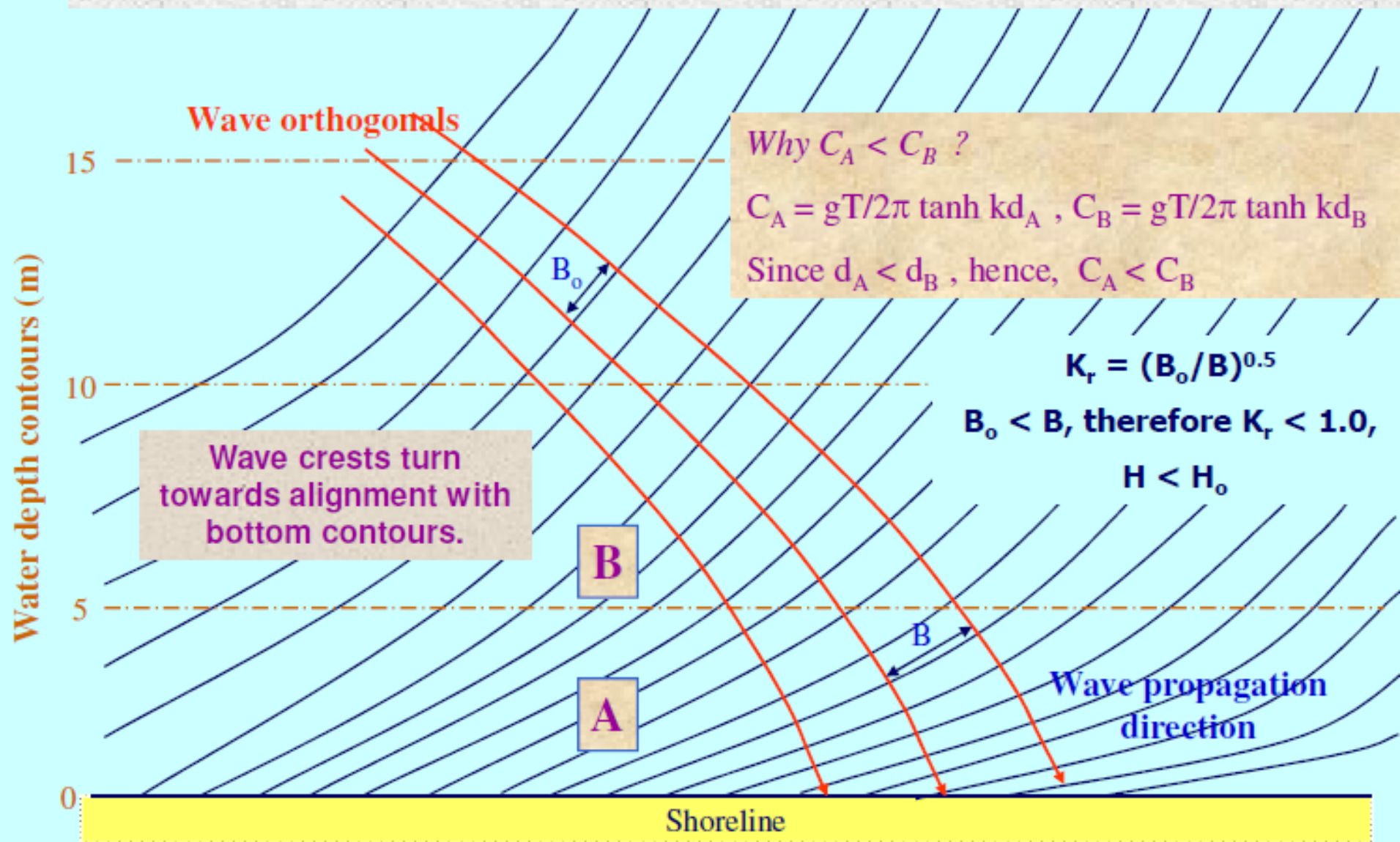
Where there are points or promontories projecting into the sea, wave fronts on both sides turn toward the point. A greatly increased amount of wave energy will be focused toward the point, and will tend to wear it away over time.



WAVE ENERGY DISTRIBUTION

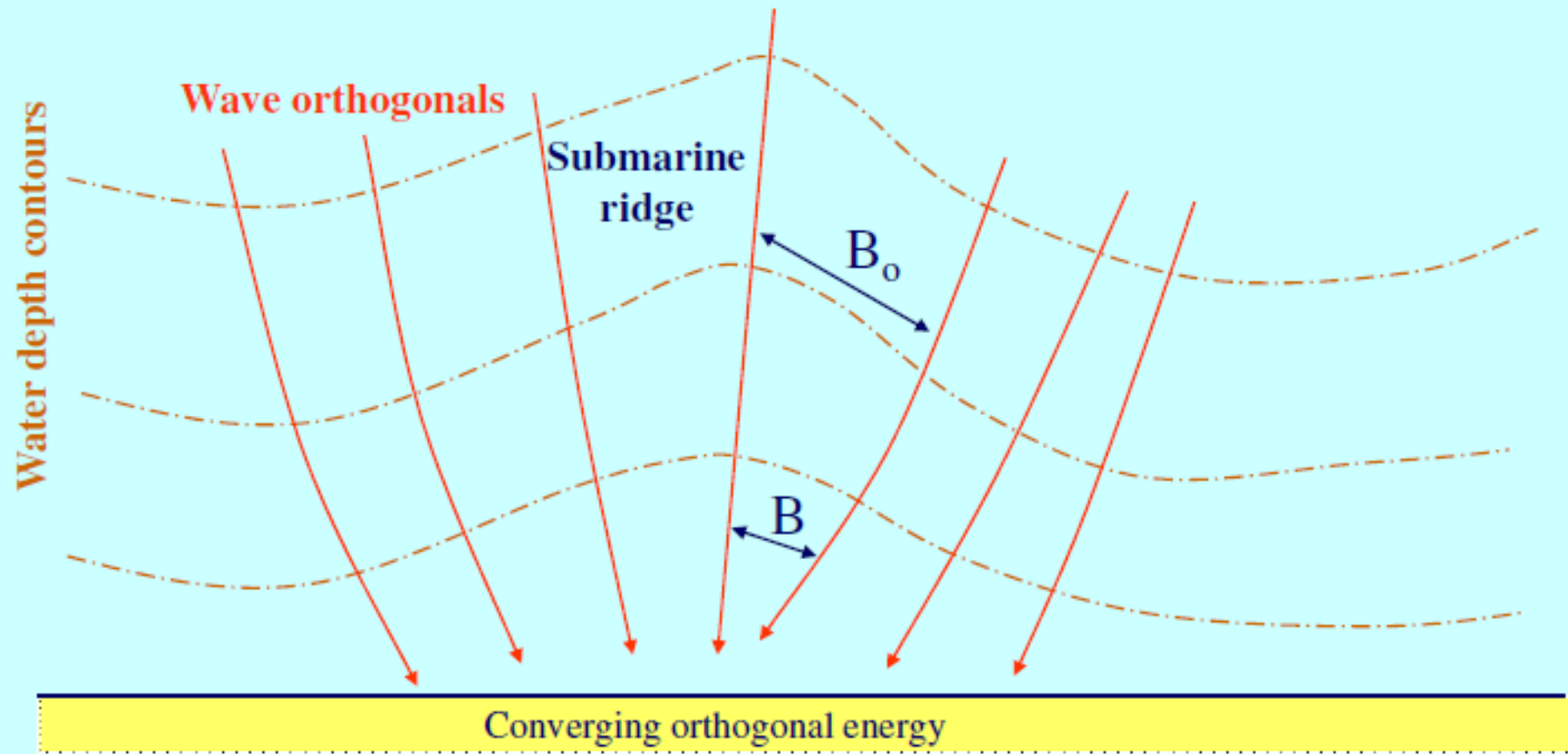


Refraction along a straight beach with parallel bottom contours



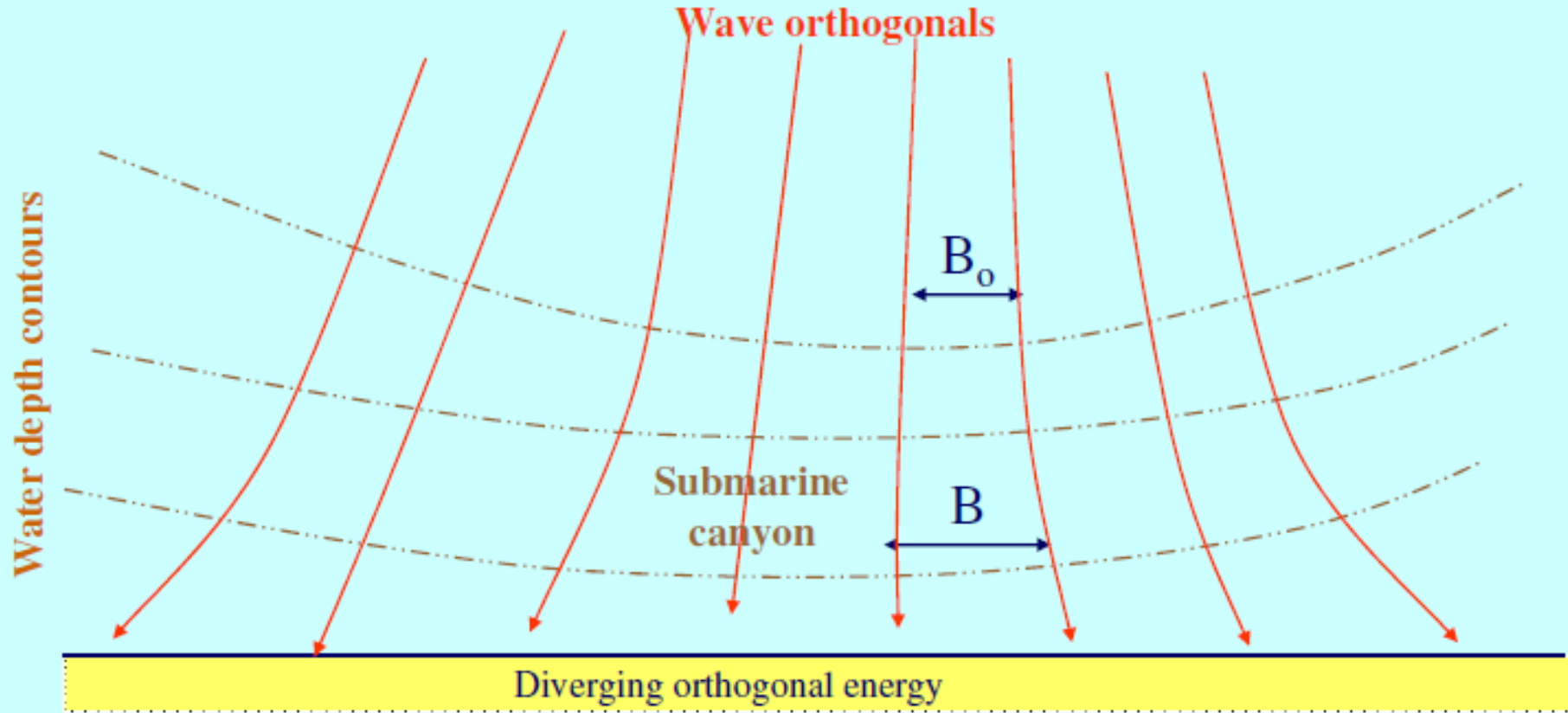
Equal distribution of wave energy along the shoreline

Refraction by a submarine ridge



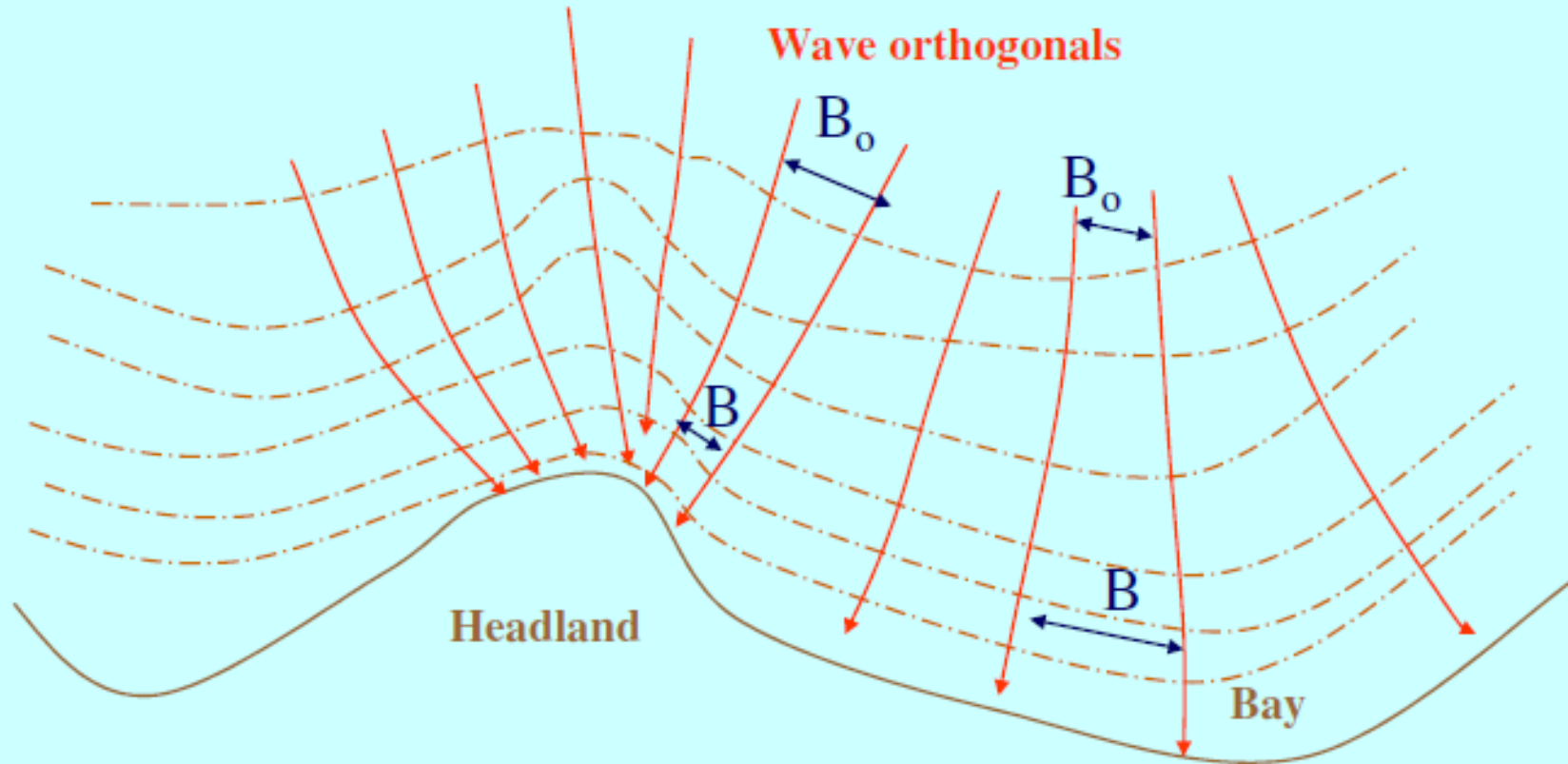
- Wave energy is concentrated due to the submarine ridge
- $B_0 > B$, therefore $K_r = (B_0/B)^{0.5} > 1.0$, $H > H_0$

Refraction by a submarine canyon



- Wave energy is diverted due to the submarine canyon
- $B_0 < B$, therefore $K_r = (B_0/B)^{0.5} < 1.0$, $H < H_0$

Refraction along an irregular shoreline



- Headland \rightarrow submarine ridge \rightarrow converging rays $\rightarrow H > H_0$
- Bay \rightarrow submarine canyon \rightarrow diverging rays $\rightarrow H < H_0$
- Wave heights are higher at a headland than in a bay



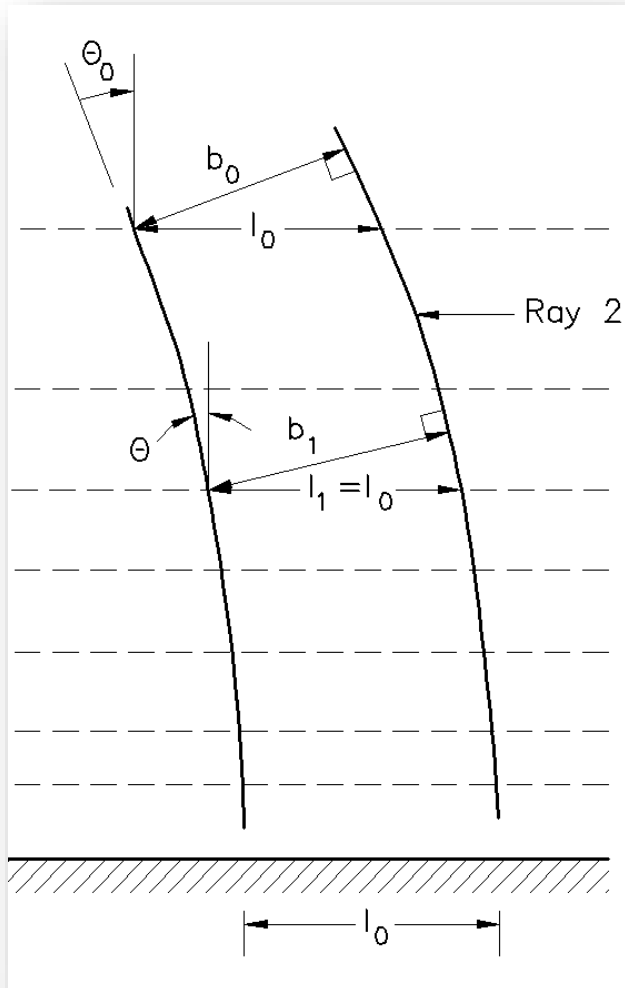
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Wave refraction analysis provides:

- Pattern of wave transformation from deepwater to shallow water.
- Determination of the near-shore wave properties and the energy distribution along the coast.



<https://twitter.com/pickmeupal/status/460442008843468800>



Consider the principle of energy conservation, the power transmitted forward between the two orthogonal is assumed to be constant, so that:

$$P_o = P$$

$$E_o C_{go} = E C_g$$

$$\frac{\rho g H_o^2 b_o}{8} \cdot C_{go} = \frac{\rho g H^2 b}{8} \cdot C_g$$

$$\frac{H}{H_o} = \left(\frac{C_{go}}{C_g} \right)^{0.5} \left(\frac{b_o}{b} \right)^{0.5} = \left(\frac{C_{go}}{C_g} \right)^{0.5} \left(\frac{\cos \alpha_o}{\cos \alpha} \right)^{0.5} = K_s K_r$$

K_r = Refraction Coefficient

If $K_r = 1$ (No refraction), $K_s = H/H_o$

Refracted wave height, H is given by

$$\frac{H}{H_o} = K_s K_r$$

H_o = Refracted deepwater wave height

K_s = Shoaling coefficient

K_r = Refraction coefficient

$$K_s = \frac{H}{H_o}$$

$$K_r = \sqrt{\frac{B_o}{B}} = \sqrt{\frac{\cos \alpha_o}{\cos \alpha}} = \left(\frac{1 - \sin^2 \alpha_o}{1 - \sin^2 \alpha} \right)^{\frac{1}{4}}$$

A wave in deep water has the following characteristics:

$$H_o = 3 \text{ m}, T = 8 \text{ s}, m = 0.02 \text{ and } \alpha_o = 30^\circ$$

Calculate refracted wave height in 10 m of water depth.

$$\frac{H}{H_o} = K_s K_r$$

$$K_s = \frac{H}{H_o'}$$

$$\frac{C}{C_o} = \tanh \frac{2\pi d}{L} = \frac{\sin \alpha}{\sin \alpha_o} = \frac{L}{L_o}$$

$$K_r = \sqrt{\frac{B_o}{B}} = \sqrt{\frac{\cos \alpha_o}{\cos \alpha}} = \left(\frac{1 - \sin^2 \alpha_o}{1 - \sin^2 \alpha} \right)^{\frac{1}{4}} = \frac{H'_o}{H_o}$$

LINEAR WAVE THEORY - EQUATIONS



RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T \sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right]$	$C_g = C = \frac{gT}{4\pi}$
5. Water Particle Velocity			
(a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh [2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	
(b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh [2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	
6. Water Particle Accelerations			
(a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh [2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	
(b) Vertical	$a_z = -2H \left(\frac{\pi}{T} \right)^2 \left(1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh [2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \cos \theta$
7. Water Particle Displacements			
(a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh [2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$\zeta = \frac{H}{2} \left(1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh [2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos \theta$
8. Subsurface Pressure	$p = \rho g (\eta - z)$	$p = \rho g \eta \frac{\cosh [2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Determine α .
 $L_0 = 9.81 \times 8^2 / [2 \times 3.142] = 99.91 \text{ m}$
 $d/L_0 = 10/99.91 = 0.1000$
 $d/L = ?$ (SPM, Table C-1)

TABLE C-1 (SPM, PP. C-8)



Table C-1. Continued.

d/L_o	d/L	$2\pi d/L$	$\frac{\text{TANH}}{2\pi d/L}$	$\frac{\text{SINH}}{2\pi d/L}$	$\frac{\text{COSH}}{2\pi d/L}$	H/H_o'	K	$4\pi d/L$	$\frac{\text{SINH}}{4\pi d/L}$	$\frac{\text{COSH}}{4\pi d/L}$	n	c_G/c_o	$\%$
.1000	.1410	.8858	.7093	1.006	1.4187	.9327	.7049	1.772	2.855	3.025	.8103	.5747	9.808

$d/L = ?$ (SPM, Table C-1)

$d/L = 0.1410$

$L = 10/0.1410 = 70.92 \text{ m}$

$$\tanh \frac{2\pi d}{L} = \frac{\sin \alpha}{\sin \alpha_o} = \frac{L}{L_o} \quad K_s = \frac{H}{H_o'}$$

$$\frac{H}{H_o} = K_s K_r$$

Determine α :

$$\sin \alpha / \sin 30^\circ = 0.7093$$

$$\alpha = 20.77^\circ$$

$$K_r = \sqrt{\frac{B_o}{B}} = \sqrt{\frac{\cos \alpha_o}{\cos \alpha}}$$

Determine K_r

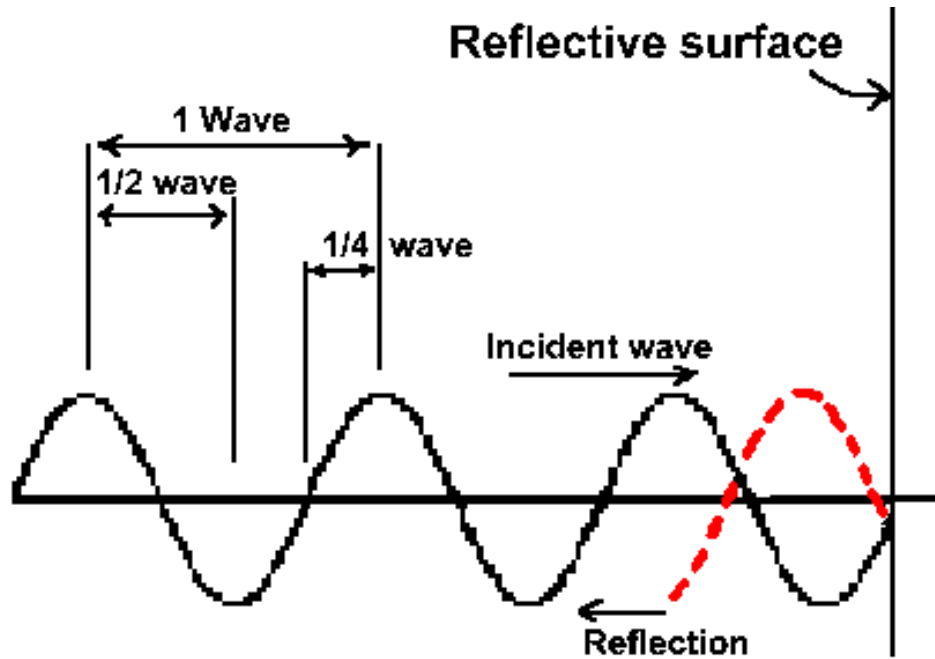
$$K_r = [\cos 30^\circ / \cos 20.77^\circ]^{0.5} = 0.9622$$

$$H = 0.9327 \times 0.9622 \times 3 = \underline{2.69 \text{ m}}$$

At the end of this lesson, students should be able to:

- understand the fundamental of wave reflection
- estimate the reflected wave height from a sloping structure.





Reflection in front of the seawall at Port Cawl, UK

When a wave hits a vertical, impermeable, rigid surface wall, **ALL** of the wave energy will essentially reflect from the wall.

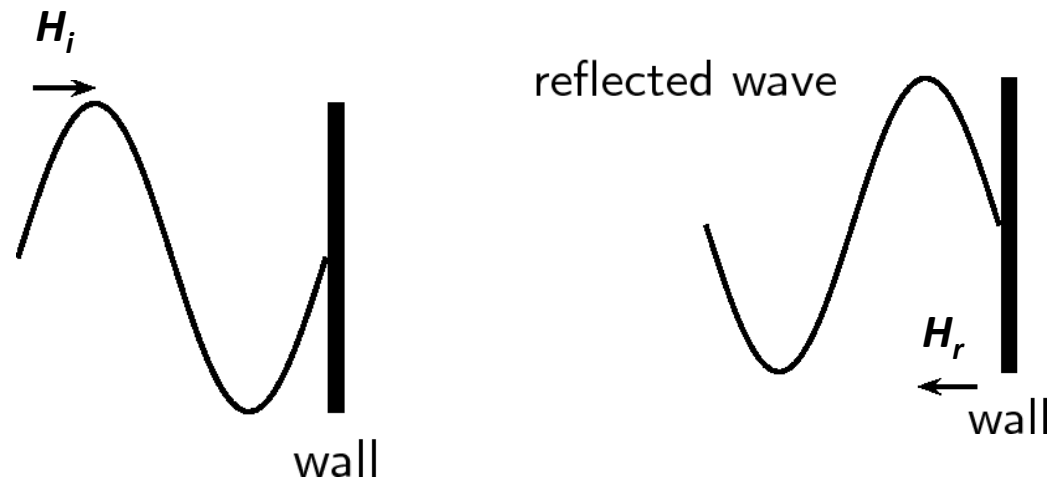
Wave motion in front of a **perfectly reflecting vertical wall** subjected to **monochromatic waves** moving in a direction perpendicular to the barrier can be determined by superimposing two waves with **identical wave numbers, periods and amplitudes** but traveling **in opposite directions**.

The water surface of the incident wave is given to a linear approximation by

$$\eta_i = \frac{H_i}{2} \cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

and the reflected wave by

$$\eta_r = \frac{H_r}{2} \cos\left(\frac{2\pi x}{L} + \frac{2\pi t}{T}\right)$$



Consequently, the water surface is given by the sum of η_i and η_r .

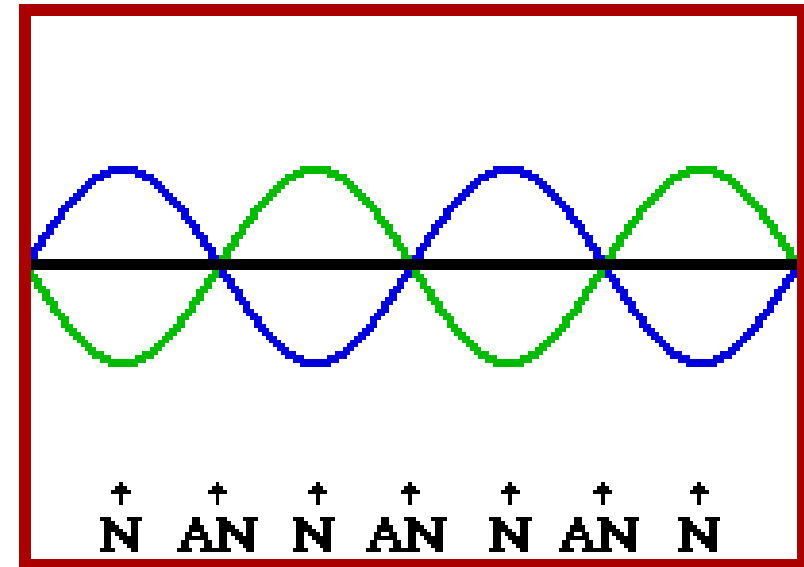
Since $H_i = H_r$,

$$\eta = \eta_i + \eta_r = \frac{H_i}{2} \left[\cos\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right) + \cos\left(\frac{2\pi x}{L} + \frac{2\pi t}{T}\right) \right]$$

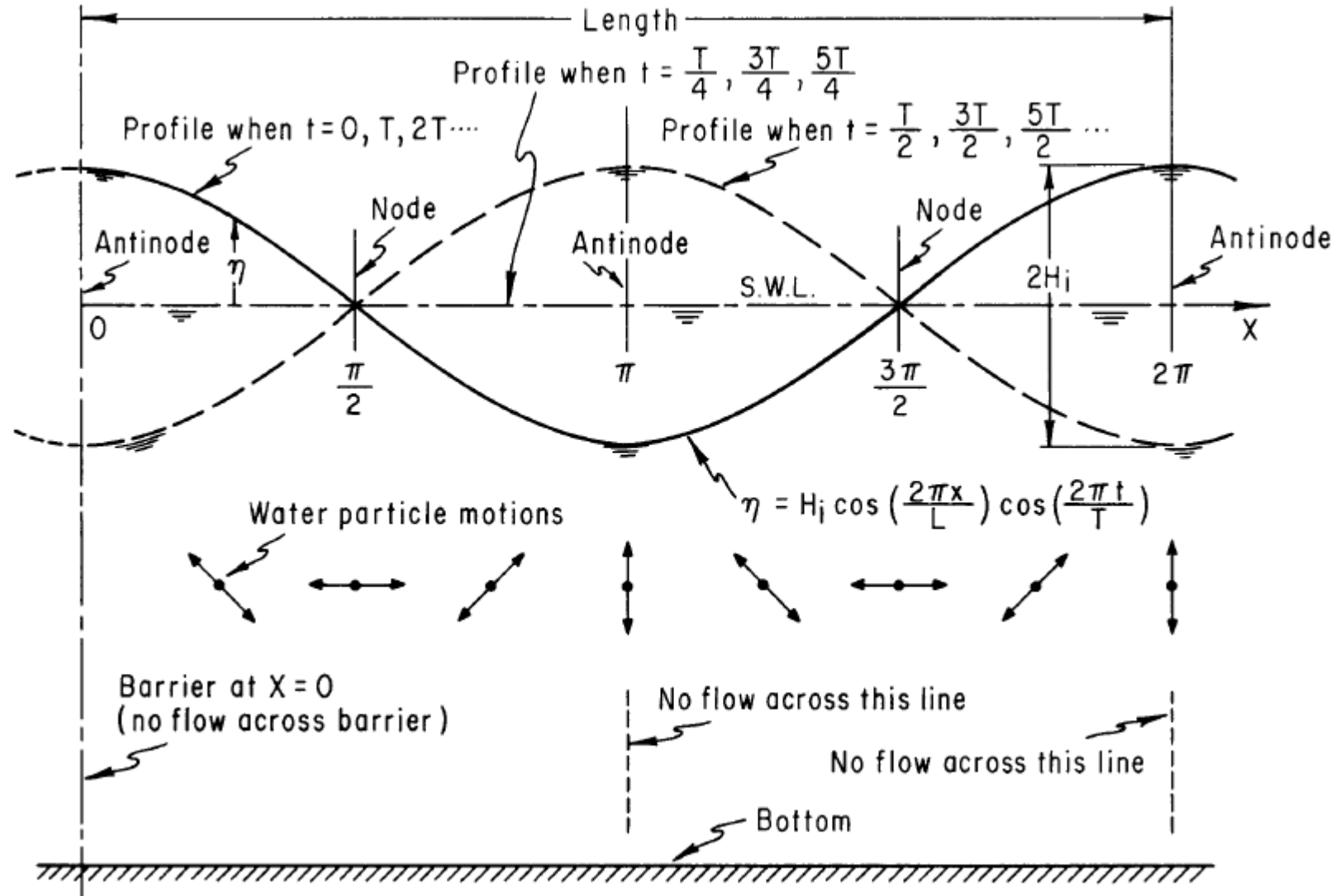
which reduces to

$$\eta = H_i \cos\frac{2\pi x}{L} \cos\frac{2\pi t}{T}$$

This equation represents the water surface for a **standing wave** or **clapotis** which is periodic in time having a **maximum height of $2H_i$** ,



STANDING WAVES (CLAPOTIS) SYSTEM



MAKING STANDING WAVES



<https://youtu.be/E9UJdITQQI>

CLAPOTIS WAVES

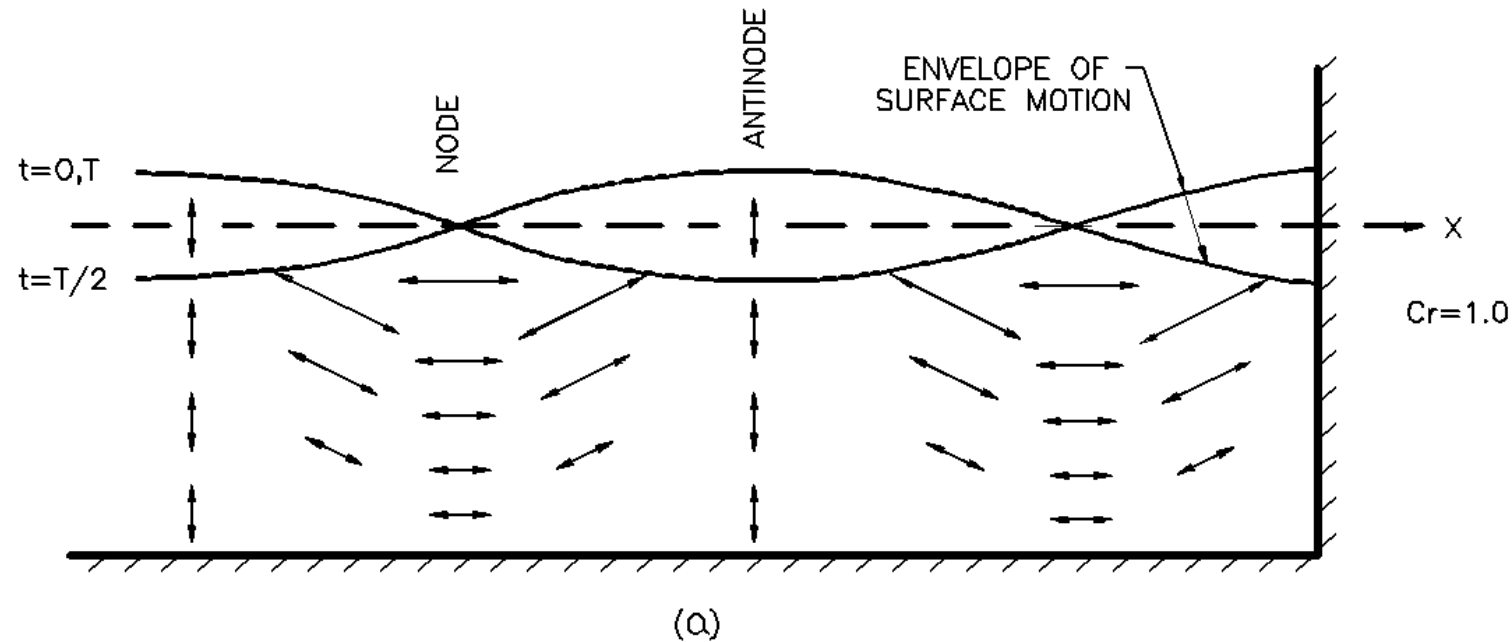


The degree of wave reflection is defined by the reflection coefficient, C_r

$$C_r = \frac{H_r}{H_i}$$

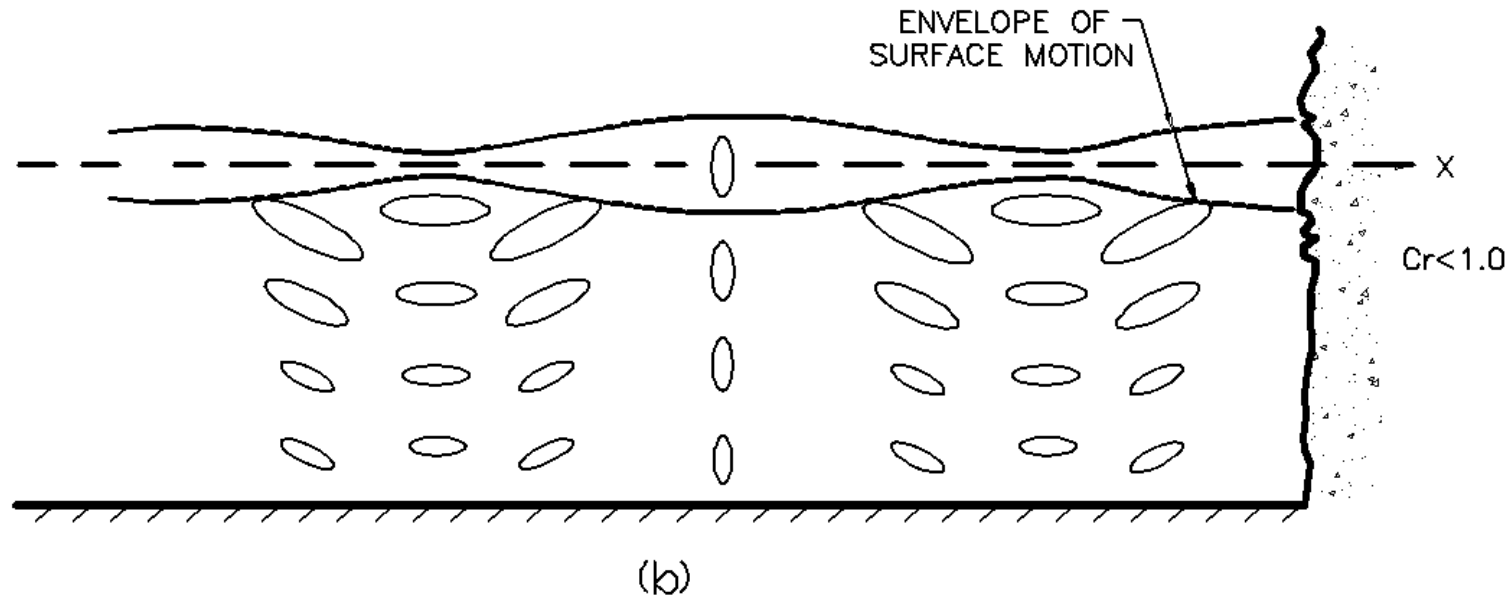
where H_r is the reflected wave heights, and H_i is the incident wave height

$C_r > 1$	\Rightarrow	Total reflection
$0 < C_r < 1$	\Rightarrow	Partial reflection
$C_r = 0$	\Rightarrow	No reflection



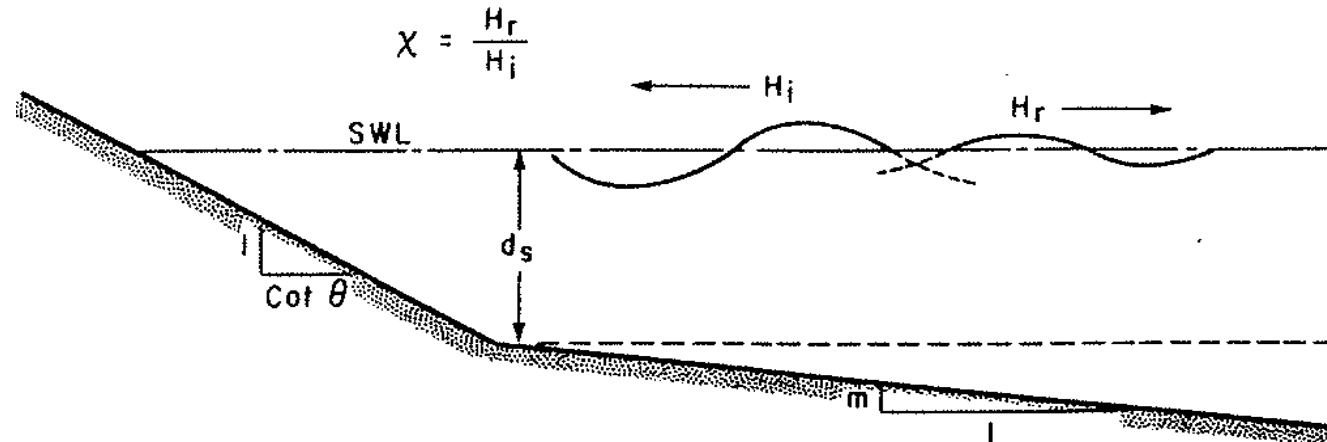
$C_r = 1:$

- At **nodes**, water particle motions are **horizontal** and all of the wave energy is **kinetic energy**.
- At **antinodes**, water particle motions are **vertical** and all of the wave energy is **potential energy**.



$C_r < 1$:

- When the reflection coefficient is less than unity, the **water surface envelope** develops.
- As the reflection coefficient decreases toward zero, the water surface profile and water particle path changes toward the form of a **normal progressive wave**.



The reflection coefficient for a reflective object depends on the **slope angle θ** , **surface roughness**, **porosity** and the **incident wave steepness H_i/L** .

Wave reflection is a function of the **Iribarren number** (Battjes 1974):

$$I_r = \frac{\tan \theta}{\sqrt{H_i / L_o}} = \frac{m}{\sqrt{H_i / L_o}}$$

ϑ = the angle the slope from the horizontal

H_i = the local incident wave height

L_o = the deepwater wavelength

The reflection coefficients for most structure forms can be given by the following:

$$C_r = \frac{aI_r^2}{b + I_r^2}$$

where the values of coefficients a and b depend primarily on the **structure geometry** and **the wave type** (i.e. monochromatic or irregular).

Table II-7-1
Wave Reflection Equation Coefficient Values Structure

Structure	a	b
Plane slope-monochromatic waves	1.0	5.5
Plane slope-irregular waves	1.1	5.7
Rubble-mound breakwaters ¹	0.6	6.6
Dolos-armored breakwaters - monochromatic waves	0.56	10.0
Tetrapod-armored breakwaters - irregular waves	0.48	9.6

¹This is an average conservative value. Seelig and Ahrens (1981) recommend a range of values for a and b that depend on the number of stone layers, the relative water depth (d/L), and the ratio of incident wave height to breaker height.

A wave in deepwater has a height of 1.8 m and a period of 6 s. It propagates towards shore without refracting or diffracting to reflect from a rubble-mound breakwater located in water 5 m deep. The breakwater slope is 1:1.75. Find the height of the reflected wave.

Given: $H_0' = 1.8$ m; $T = 6$ s; $d = 5$ m; $\vartheta = \tan^{-1}(1/1.75) = 29.74^\circ$

$$L_0 = gT^2/2\pi = 56.21 \text{ m}$$

$$d/L_0 = 5/56.21 = 0.0890$$

From Table C-1: $d/L = 0.1313$ (transitional water); $H/H_0' = 0.9433$

[Note: H = the local wave height at $d = 5$ m]

$$H = 0.9433 \times 1.8 = 1.6979 \text{ m} = 1.7 \text{ m}$$

$$I_r = \frac{\tan \theta}{\sqrt{H_i/L_0}} = \frac{m}{\sqrt{H_i/L_0}} \quad I_r = \frac{\tan 29.7^\circ}{\sqrt{1.70/56.2}} = 3.28$$

$a = 0.6$ and $b = 6.6$ (from Table II-7-1).

$$C_r = \frac{aI_r^2}{b + I_r^2}$$

$$C_r = \frac{0.6(3.28)^2}{6.6 + (3.28)^2} = 0.37$$

the reflected wave height $H_r = C_r H_i = 0.37(1.70) = 0.63$ m.

$$C_r = \frac{aI_r^2}{b + I_r^2}$$

Table II-7-1
Wave Reflection Equation Coefficient Values Structure

Structure	a	b
Plane slope-monochromatic waves	1.0	5.5
Plane slope-irregular waves	1.1	5.7
Rubble-mound breakwaters ¹	0.6	6.6
Dolos-armored breakwaters - monochromatic waves	0.56	10.0
Tetrapod-armored breakwaters - irregular waves	0.48	9.6

¹This is an average conservative value. Seelig and Ahrens (1981) recommend a range of values for *a* and *b* that depend on the number of stone layers, the relative water depth (*d/L*), and the ratio of incident wave height to breaker height.

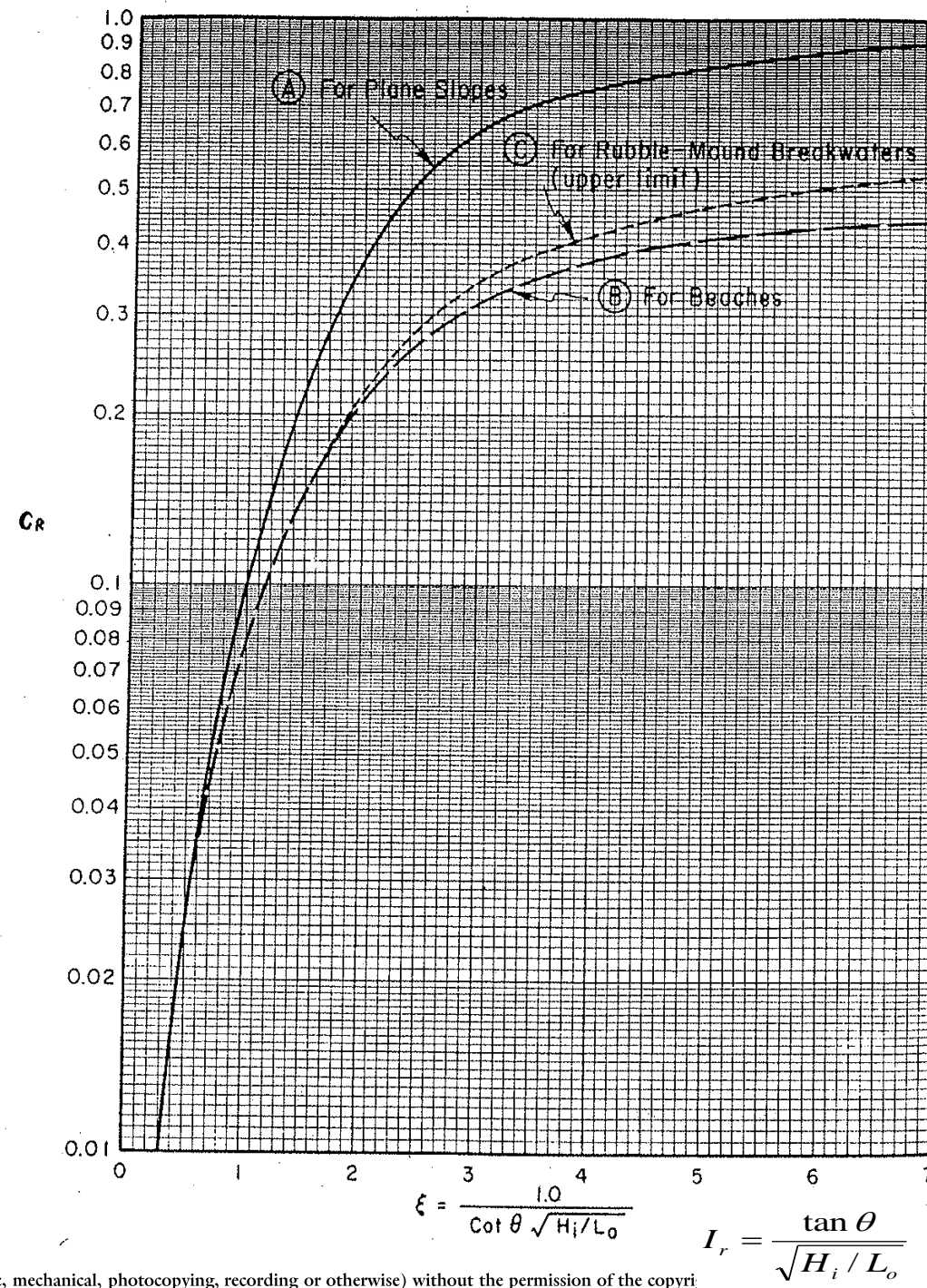
When the reflecting slope becomes very flat, the incident wave will break on slope. This causes increase in energy dissipation and decrease in reflection coefficient. Thus, beaches are generally very efficient wave absorbers, particularly for shorter period wind waves.

Seelig and Ahrens (1981) suggest that $a = 0.5$ and $b = 5.5$ be used for beaches. Since the slope angles are small, the Iribarren number I_r will be relatively small, yielding relatively low reflection coefficients.

$$C_r = \frac{aI_r^2}{b + I_r^2}$$

Based on a compilation of measurements from several sources, Seelig and Ahrens (1981) developed the curves used to obtain a high estimate of C_r for:

- (a) smooth slope
- (b) sand beaches
- (c) rubble-mound breakwaters



The curves show C_r decreases as either

- (a) the wave steepness increases,
- or
- (b) the slope angle ϑ decreases

PROBLEM 2

GIVEN: An incident wave with period $T = 10$ seconds and a wave height $H_1 = 2$ meters (6.56 feet) impinges on a slope.

FIND:

- (a) The height of the wave reflected from an impermeable slope with $\cot\theta = 5.0$.
- (b) Compare the reflection coefficient obtained in (a) above with that obtained for a beach with $\cot\theta = 50$.

Problem 2: Solution

SOLUTION: Calculate

$$(a) \quad L_o = \frac{gT^2}{2\pi} = \frac{9.8(100)}{2\pi} = 156 \text{ m (512 ft)}$$

and from equation (2-86)

$$I_r = \frac{\tan \theta}{\sqrt{H_i/L_o}} = \frac{1}{\cot \theta \sqrt{H_i/L_o}} \quad \xi = \frac{1.0}{5.0 \sqrt{2/156}} = 1.77$$

The reflection coefficient from curve A for plane slopes in Figure 2-65 is $\chi = 0.29$; therefore, the reflected wave height is $H_r = 0.29(2) = 0.58$ meter (1.90 feet).

$$\Delta = \frac{H_r}{H_i}$$

(b) For a 1 on 50 sloped beach,

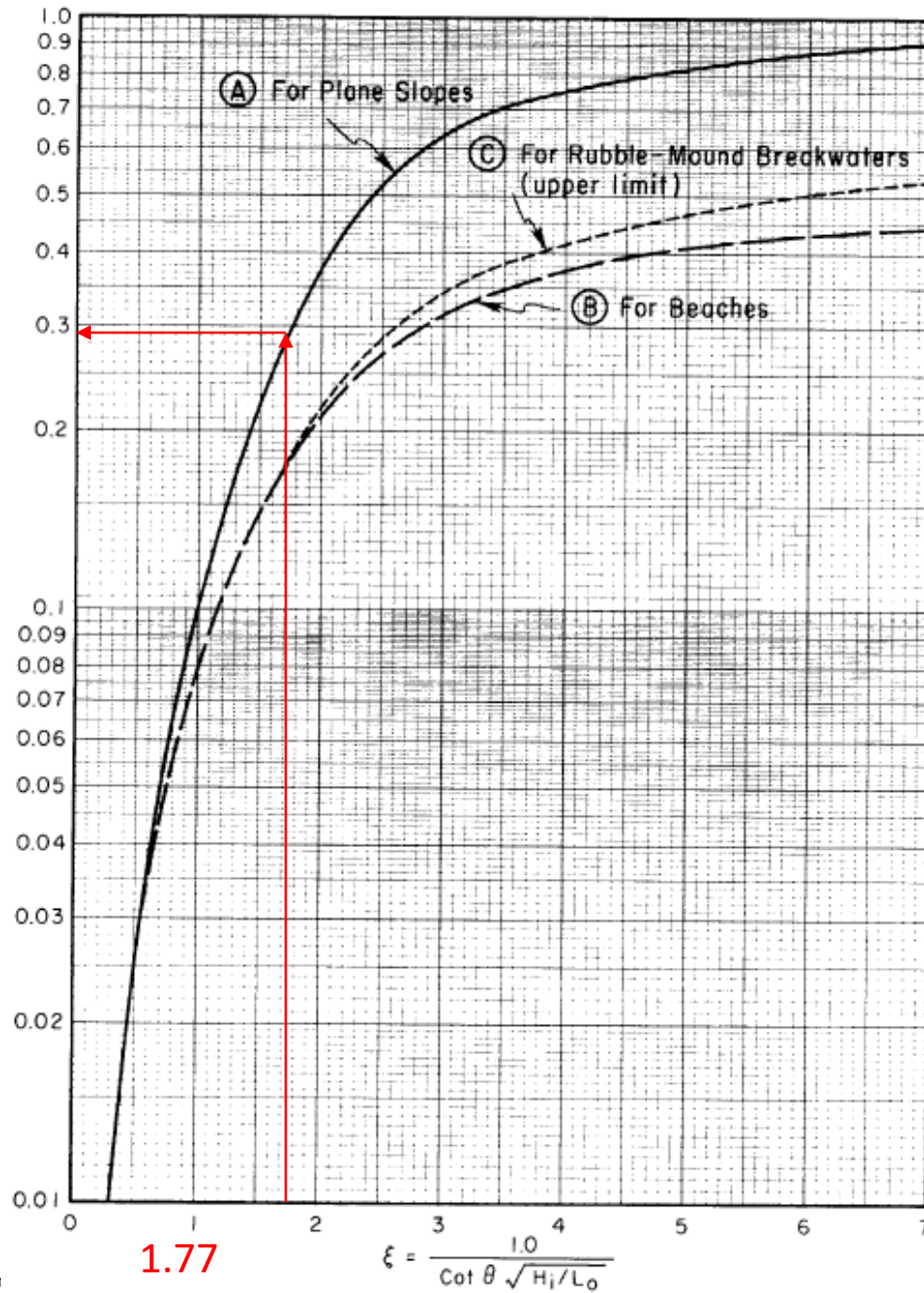
$$\xi = \frac{1.0}{50.0 \sqrt{2/156}} = 0.18$$

From curve B in Figure 2-65, $\chi < 0.01$ for the beach. The 1 on 50 beach slope reflects less wave energy and is a better wave energy dissipater than the 1 on 5 structure slope.

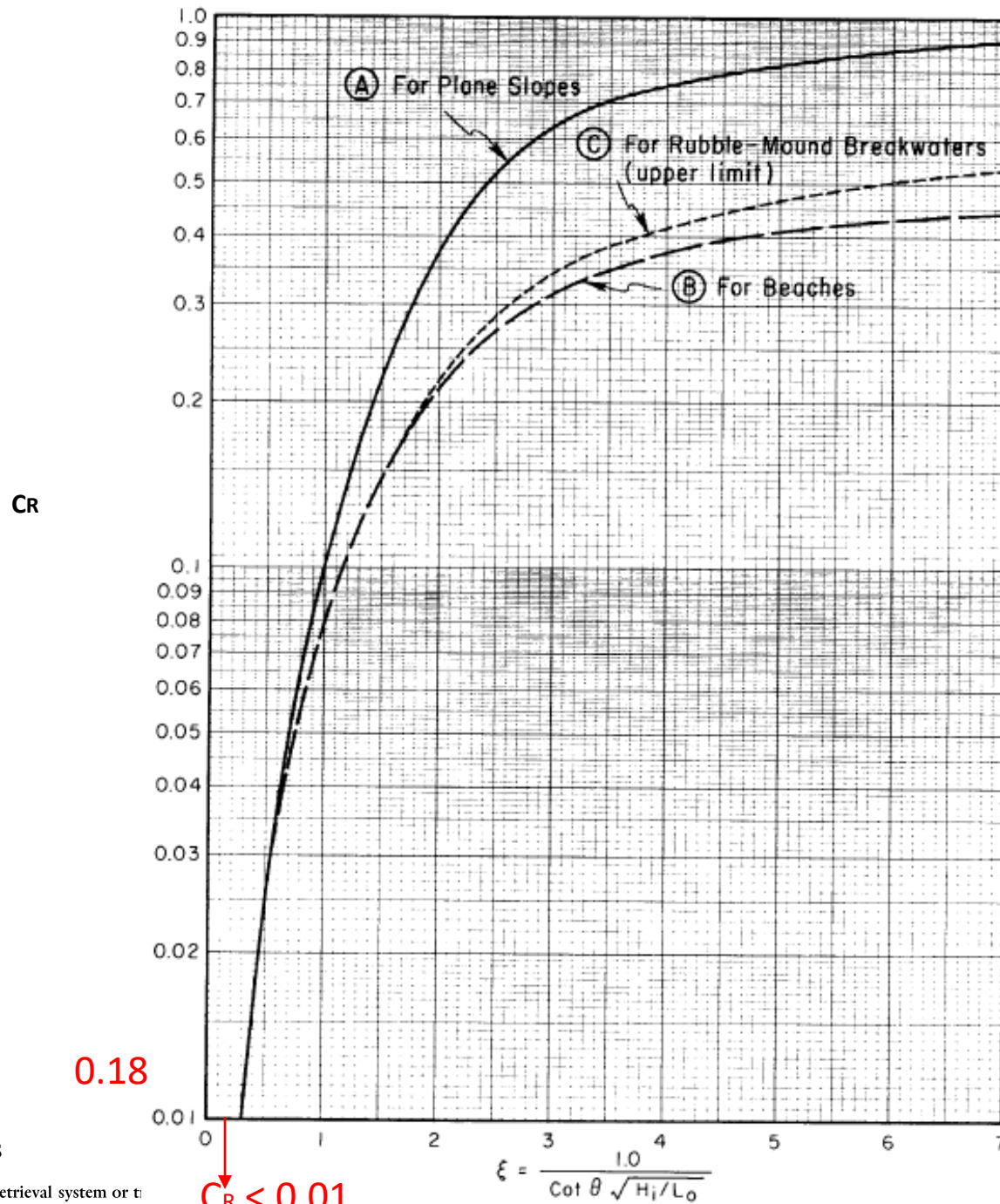
Q(a)

0.29

CR



Q(b)



PROBLEM 3

GIVEN: Waves with a height $H_1 = 3.0$ meters (9.84 feet) and a period $T = 7$ seconds are normally incident to a rubble-mound breakwater with a slope of 1 on 2 ($\cot\theta = 2.0$).

FIND: A high estimate (upper bound) of the reflection coefficient.

Solution - Problem 3

SOLUTION: Calculate

$$L_o = \frac{gT^2}{2\pi} = \frac{9.8(7.0)^2}{2\pi} = 76.4 \text{ m (251 ft)}$$

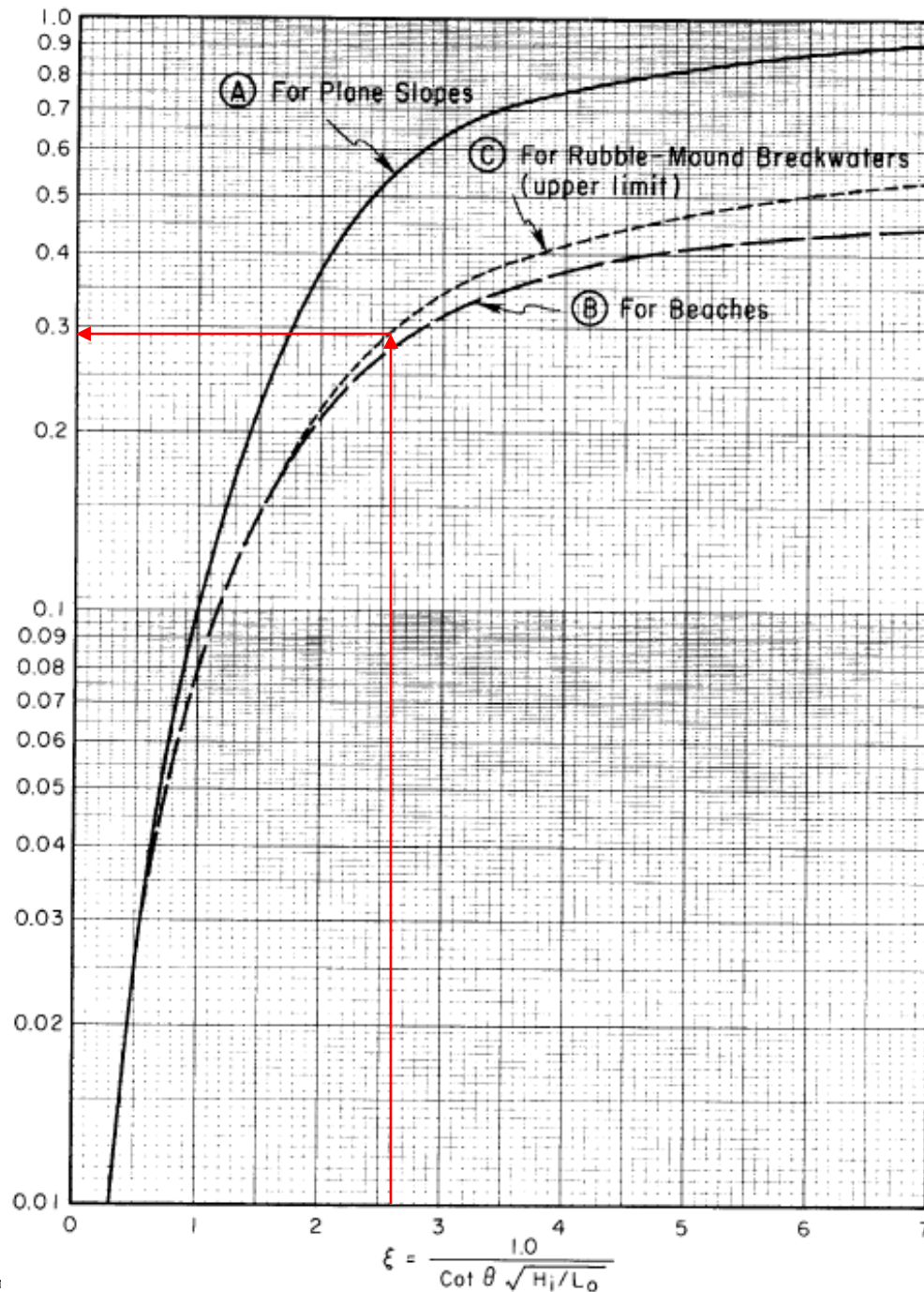
and from equation (2-86)

$$\xi = \frac{1.0}{2.0 \sqrt{3.0/76.4}} = 2.52$$

From curve C in Figure 2-64, $\chi = 0.29$ which is the desired upper bound on χ . The actual reflection coefficient depends on wave transmission, internal dissipation, overtopping, and many other factors. Techniques described in Seelig and Ahrens (1981) and laboratory tests by Seelig (1980) should be used to obtain better wave reflection coefficient estimates for breakwaters.

0.29

CR





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